Binary Stars - astro8501 - 6944

Problem Sheet 8

1. In the lecture we derived a simple formula for $\zeta_L = \frac{\partial \ln R_L}{\partial \ln M}$, the exponent for the Roche lobe change when mass is lost (or gained),
$$\zeta_L = 2.13\frac{M_1}{M_2} - 1.67 = \frac{2.13}{q} - 1.67$$
where $q = M_2/M_1$ (star 1 is the donor, star 2 the accretor). The analysis can be improved as follows (following Soberman et al. 1997). Assume that only a fraction of the mass β is accreted on the secondary, i.e.
$$dM_2 = -\beta dM_1$$
and that the mass which leaves the binary carries angular momentum
$$dJ = \gamma J \frac{d\left(M_1 + M_2\right)}{M_1 + M_2}.$$
Derive an expression for ζ_L where $R_L = af(q)$ ($q = M_2/M_1$ and $f(q)$ is given by some fitting formula, the derivatives of which you are not expected to calculate). What is ζ_L for $\beta = 0, 1$ and $\gamma = 1$? What do these values of β and γ physically represent?

2. Derive a_f/a_i for the α and γ common-envelope prescriptions, where
$$\alpha = \frac{\Delta E_{\mathrm{bind}}}{\Delta E_{\mathrm{orb}}}$$
and
$$\frac{\Delta J}{J} = \gamma \frac{\Delta M}{M},$$
where ΔE is the change in energy (binding or orbital respectively), ΔJ is the change in angular momentum and ΔM the change in mass during the common envelope phase. Assume $\Delta E_{\mathrm{bind}} = GMM_{\mathrm{env}}/\lambda R$.

3. Given results of Q2, estimate the final separation after common envelope evolution in a binary containing two solar-mass stars, one of which has a core mass $0.5\,M_\odot$ when it overflows its Roche Lobe (i.e. $M_1 = M_2 = 1\,M_\odot$, $R = 100\,R_\odot$) . Assume $\alpha = 0.2$ and $\lambda = 0.5$ (the latest papers suggest $\alpha = 0.2$). Repeat for the γ prescription with $\gamma = 1.5$ (as suggested by Nelemans and Tout 2005). What fraction of the orbital energy is lost in this case and the equivalent α? Given that wide binaries with white dwarf companions exist at periods which suggest they cannot have avoided RLOF, what does this tell you?

4. Estimate a minimum timescale (in years) for common envelope evolution that leads to a close WD-WD pair, given that the orbital energy release rate cannot exceed the Eddington luminosity.

5. Estimate the range of orbital periods for which common envelope evolution will occur in a binary containing a $3\,M_\odot$ thermally-pulsing AGB star and a solar-like star.

Questions, problems, errors? Contact Rob Izzard by email: izzard@astro.uni-bonn.de