UniS

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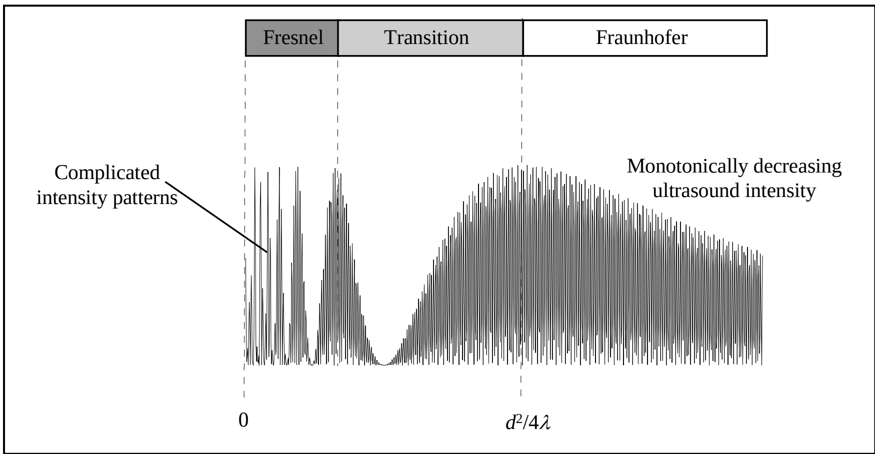
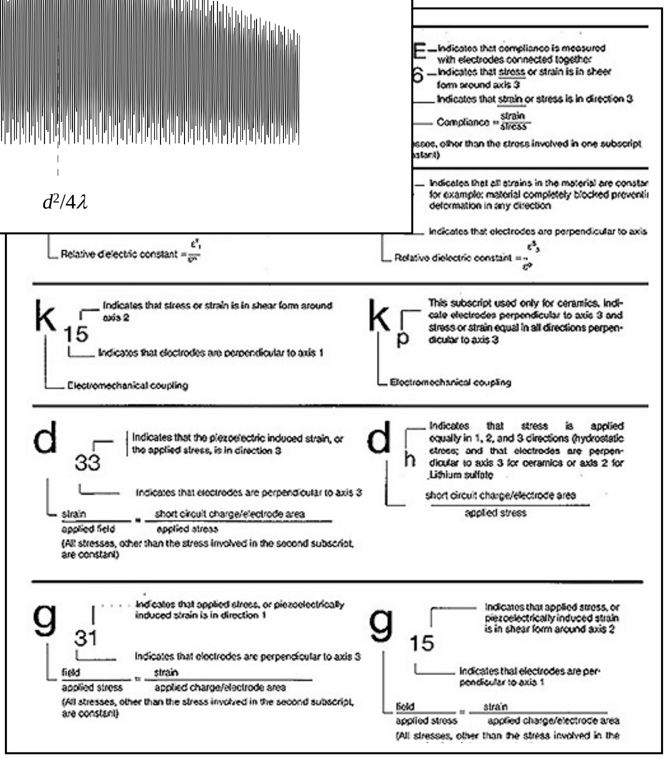
Level 2: Ultrasonics





Academic Year 1998–1999

Lecturer and Course Developer Dr. S. J. Doran

ULTRASONICS (2US)

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Course modified and lectured by Dr. S. J. Doran

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Contents

Chapter 1

Introduction

1.1 Topics to be covered

- Wave propagation in liquids and solids - plane waves, spherical waves.
- Speed of waves - longitudinal, transverse, surface and plate waves.
- Reflection and transmission at boundaries, characteristic acoustic impedance, mode conversion.
- Attenuation of acoustic signals by absorption, diffraction and scattering.
- Piezoelectric materials.
- Transducers and hydrophones - their construction and use. Plane piston and focused transducers - field patterns, near and far field. Directivity properties of hydrophones.
- Applications of ultrasound in solid state physics, physical chemistry, medicine, non-destructive testing and underwater acoustics.
- Calibration of transducers and hydrophones. Measurement uncertainties.
- How to recognise non-linear effects in acoustic measurements.

1.2 Aims

This course is designed to be complementary to the Level 2 Medical Acoustics course. It will focus on a number of largely non-medical applications of ultrasonics and will aim to do the following things:

- to provide a practical introduction to aspects of ultrasound which may be of assistance to students about to undertake their industrial training year;
- to revise the fundamental features of wave propagation;
- to give students confidence in making use of simple models of transducer and hydrophone behaviour;
- to indicate the limitations of some theoretical approaches to the modelling of the propagation of ultrasound;
- to give students practice in extracting relevant information from published sources.

We will be avoiding the lengthy derivation of acoustic wave equations but will concentrate on using the results of such derivations and on understanding the assumptions on which the results rely.

1.3 Assessment

70% by examination:
The standard Level 2 structure — 1 hour exam; choose two of three questions

30% by coursework:
The form of this element has changed this year. There will be a mix of two different elements. 40% of the marks will be awarded for essay style answers (each about 500 words) to Revision Questions 1 and 15 from the booklet. The

remaining 60% of the coursework mark will be for a project, details of which are given below and will be explained during the first lecture.

1.4 Project

The Project is a mini literature research project, whose subject will be one of the applications of ultrasonics technology below. The results of which are to be presented in the form of a scientific review article: that is it should have an Abstract, Introduction, Theory Section (where relevant), Discussion of Current Applications and Conclusions.

This coursework is analagous to a lab report that you might do in a practical module. In it you can include material from the Internet, including pictures taken from Web pages. However, I do not want just a cobbled together version of somebody else's Web pages; you need to provide *your own* text and analsysis of the information available. The report should be no longer than 20 sides of A4, including all diagrams and appendices. All information (quotes, theory, pictures, etc.) *must* be referenced. For the computer hackers among you, I will also accept the report in HTML, as long as it's length equates to being less than 20 sides of A4.

Four different projects will be available:

- Ultrasound flow measurements
- Sonar
- Non-destructive testing
- Piezoelectric transducers and hydrophones

The handing in deadline for the project will be the Friday of Week 4 (i.e., Friday, 12th February. After reading the reports, I will allocate (if all goes well on the lecture of Monday 15th February) each of you a section of your report to talk about duuring the remaining lectures. And you should all have the opportunity to do a brief 5 – 10 minute presentation to the rest of the class.

1.5 Sources of Reference

It is not expected that you should have to buy any textbook for this course. The exam material will be based upon these notes and additional material given during the lectures. A key feature of the course is the considerable use of Ultrasonics material freely available on the Internet. *You are expected to undertake* **active** *research for your projects using the World Wide Web.* Below is a set of URL's to provide you with a start, but there is plenty more around. The bookmarks below are available in a special file `us_bookmarks.html` in the directory `$phlib`, so you don't even have to type them in.

Ultrasound flow measurements

`http://intersoft-electronics.com/sport/windspeed/windspeed.htm`

`http://intersoft-electronics.com/sport/windspeed/windspeed.htm`

`http://www.signal-processing.com/techinf.htm`

`http://www.aoml.noaa.gov/phod/adcp/`

`http://www.dhi.dk/project/0instr/adcp/adcp.htm`

`http://www-dinind.er.usgs.gov/nawqa/wr03019.htm`

`http://marine.rutgers.edu/mrs/leodata.html`

`http://www.bath.ac.uk/Departments/Physics/unde1b.html`

`http://web1.pipemedia.net/katronic/products.htm` http://www.physiology.com/cbi202.htm

http://www.crh.noaa.gov/cys/inst.htm

Sonar

http://www.dra.hmg.gb/html/products/seasys/ssohasn.htm

http://www.ifremer.fr/genavir/engins/sonar_uk.htm

http://www.cee.hw.ac.uk/ ceedjd/project.html

http://www.sio.rssi.ru/sonar_e.htm

http://www.arl.psu.edu/techareas/acspr/acspr.html

http://www.sonardyne.com/

http://www.soc.soton.ac.uk/

Non-destructive testing

http://www.irtest.com/ultrasonic.htm

http://www.ndt.net/article/imm/imm_e.htm

http://www.ndt.net/abstract/ut97/wt1097/panam.htm

http://www.ndt.net/article/ecndt98/ut/468/468.htm

http://www.ndt.net/article/ecndt98/ut/471/471.htm

http://www.ndt.net/article/ecndt98/general/497/497.htm

http://www.ndt.net/article/ecndt98/aero/002/002.htm

http://www.ndt.net/article/1198/davis/davis2.htm

http://www.ndt.net/article/ecndt98/nuclear/239/239.htm

http://www.ndt.net/article/ecndt98/aero/046/046.htm

An additional excellent reference is the document "Nondestructive material testing with ultrasonics: Introduction to the basic principles" by Krautkramer. This is stored in the PDF document format and can be found in the `$phlib/2us/krautkramer.pdf` file on the UNIX systems, which you view using Adobe Acrobat Reader 3.0 or higher.

Piezo-electric transducers and hydrophones

`http://www.piezo.com/index.html`

`http://www.panametrics.com/pci-rd/experimental/index.shtml`

`http://www.piezo.com/reading.html`

`http://gummo.stanford.edu/DD/Courses/me220/lectures/lect06/lect_6.html`

`http://www.matroc.com/mmecframe.html` (Fom here you need to do a bit of navigating through the frames: Choose "Products" in the left-hand frame, then scroll down to "Electroceramics Division, Ohio", then, finally, "Online Catalog", followed by "PZT Specifications", "PZT Ceramic Materials" or "PZT Properties")

Those undertaking this project should also obtain from me a copy of Preston *et al.*, *J. Phys. E Sci. Instrum.*, **16** 786 – 796 (1983)

But remember, there is lots more out there. You will get best marks by doing your own original searches and writing an interesting report that teaches *me* something new!

To find new material, use one of the many Internet "search engines" such as:

`http://altavista.digital.com`

`http://webcrawler.com`

`http://www.lycos.com`

You may also wish to refer to one or more of the following textbooks used in the preparation of this course:

- Cracknell A.P. (1980), Ultrasonics, Wykeham Publications (London) Ltd.
- Hull B. & John V. (1988) Non-destructive testing, Macmillan Education.
- Kaye G.W.C. & Laby T.H. (eds) (1995) Tables of physical and chemical constants, 16th edition, Longman.
- Kinsler L.F., Frey A.R., Coppens A.B. & Sanders J.V., (1982), Fundamentals of acoustics (3rd edition), John Wiley.
- Meyer E. & Neumann E-G., (1972), Physical and applied acoustics: an introduction, Academic Press.
- Morse P.M. & Ingard K.U. (1968) Theoretical acoustics, McGraw-Hill
- Pierce A.D. (1981), Acoustics: an introduction to its physical principles and applications, McGraw-Hill.
- Pope J.A. (1989) Medical physics, (2nd edition), Heinemann Educational.
- Preston R.C. (ed), (1991) Output measurements for medical ultrasound, Springer-Verlag. See Chapter 2, Fundamentals of ultrasonic propagation, by Dr. R.C. Chivers.
- Seto W.W. (1971), Theory and problems of acoustics, McGraw-Hill.
- Towne D.H. (1988), Wave phenomena, Dover.
- Lynnworth L.C. (1989) Ultrasonic measurements for process control: theory, techniques, applications, Academic Press.
- Harburn G., Taylor C.A. and Welberry T.R., (1975) Atlas of optical transforms, G. Bell & Sons Ltd, London.
- Gooberman G.L. (1968) Ultrasonics: theory and applications, English Universities Press.

- ter Haar G.R. (1986) Ultrasonic biophysics, in Physical principles of medical ultrasonics, ed. C.R. Hill, Ellis Horwood.

Finally, for the really ambitious, there are many experimental research papers in these fields. You may wish to try to find some using the search facility called BIDS, located at `http://www.bids.ac.uk`. For this you will need a suitable password. See me if interested.

Chapter 2

Propagation of acoustic waves

For background refer to Kinsler, Frey et al. (1982), chapters five and six and to Seto (1971) chapters two, three and four.

Ultrasonic frequencies are normally taken to be those above the audible range, typically greater than 20 kHz. In practice, for medical applications and non-destructive testing we will be concerned with frequencies in the low MHz range and for underwater acoustics applications the frequency range is typically up to 500 kHz. Closer to the audible range we find the frequencies used in ultrasonic cleaning baths, which may operate a few kilohertz above 20 kHz.

We will consider only two types of linear acoustic waves: plane waves and spherical waves. Some aspects of the non-linear propagation of waves will be considered at the end of the course of lectures.

2.1 Plane waves

The one-dimensional wave equation is:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \tag{2.1}$$

where p is acoustic pressure, t is time, c is the speed of sound and x is position.

This equation applies when all acoustic variables are a function of only one spatial co-ordinate (in this case, x), so that phase is constant over any plane perpendicular to the x direction. It applies equally to longitudinal or compressional waves, and to transverse or shear waves.

Important to recognise the meaning of pressure, p, in this equation. Let P_o be the constant equilibrium pressure in a fluid, for example, and let P be the instantaneous pressure at any point, Then:

$$p = P - P_o. \tag{2.2}$$

p is called the excess pressure or acoustic pressure, and it is this quantity with which we are concerned in the acoustic wave equation. Note the fact that p can take both positive and negative values.

It is often useful to consider the density of a material in a similar way. If ρ_o is the constant equilibrium density and ρ is the instantaneous density at any point, then we can define a condensation variable s, so that:

$$s = \frac{\rho - \rho_o}{\rho_o}. \tag{2.3}$$

s can also have positive or negative values. Regions where $s > 0$ are called condensations and regions where $s < 0$ are called rarefactions. In an acoustic disturbance in the absence of absorption of the acoustic energy by loss mechanisms in the medium, p and s are in phase ($p = \rho_o c^2 s$).

The solution of equation 2.1 can be written using the exponential form as:

$$\overline{p} = \overline{A}\exp[i(\omega t - kx)] + \overline{B}\exp[i(\omega t + kx)], \tag{2.4}$$

where the overbar indicates that the variable is a complex quantity, e.g. $\overline{A} = A\exp(i\phi_a)$ where ϕ_a is a phase angle.

The relationship between acoustic particle velocity and acoustic pressure, assuming linearity and ignoring viscosity, is:

$$\rho_o \frac{\partial \underline{u}}{\partial t} = -\nabla p, \tag{2.5}$$

where $\underline{u}$ is the particle velocity, a vector quantity, as indicated by the underlining.

For the plane wave we are considering, the acoustic particle velocity is:

$$\underline{u} = \left[\frac{\overline{A}}{\rho_o c} \exp[i(\omega t - kx)] - \frac{\overline{B}}{\rho_o c} \exp[i(\omega t + kx)] \right] \hat{x}, \tag{2.6}$$

where $\hat{x}$ is the unit vector in the x direction.

The wave travelling in the positive x direction is:

$$\overline{p}_+ = \overline{A} \exp[i(\omega t - kx)], \tag{2.7}$$

and the wave travelling in the negative x direction is:

$$\overline{p}_- = \overline{B} \exp[i(\omega t + kx)]. \tag{2.8}$$

Then for u_+, u_-, s_+ and s_- we find that:

$$\overline{u}_+ = +\frac{\overline{p}_+}{\rho_o c}; \tag{2.9}$$

$$\overline{u}_- = -\frac{\overline{p}_-}{\rho_o c}; \tag{2.10}$$

$$\overline{s}_+ = +\frac{\overline{p}_+}{\rho_o c^2}; \tag{2.11}$$

$$\overline{s}_- = -\frac{\overline{p}_-}{\rho_o c^2}. \tag{2.12}$$

For a plane wave travelling in an arbitrary direction we could try a solution to the wave equation of the form:

$$\overline{p} = \overline{A} \exp[i(\omega t - k_x x - k_y y - k_z z)]. \tag{2.13}$$

This is valid if

$$\frac{\omega}{c} = \sqrt{k_x^2 \hat{x} + k_y^2 \hat{y} + k_z^2 \hat{z}}. \tag{2.14}$$

If we define the vector $\underline{k}$ as

$$\underline{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}, \tag{2.15}$$

where $|\underline{k}| = \omega / c$, and use $\underline{r}$ as the position vector, so that

$$\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}, \tag{2.16}$$

we can write our solution as

$$\overline{p} = \overline{A}\exp[i(\omega t - \underline{k}\cdot\underline{r})]. \tag{2.17}$$

Surfaces of constant phase are given by $\underline{k}\cdot\underline{r} =$ constant. Because $\underline{k} = \nabla(\underline{k}\cdot\underline{r})$ is a vector perpendicular to the surfaces of constant phase, $\underline{k}$ points in the direction of propagation of the plane wave.

Question Show that $\Psi = \Psi_o \sin(kx)\cos(\omega t)$ is a solution of the one-dimensional wave equation

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}. \tag{2.18}$$

If the particle velocity distribution, $u(x,t)$, in an acoustic plane wave is given by

$$u = u_o \sin(kx)\cos(\omega t), \tag{2.19}$$

what is the corresponding pressure distribution?

2.2 Spherical waves

If we assume spherical symmetry, so that the acoustic disturbance is solely a function of radial distance and time, the acoustic wave equation becomes:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \tag{2.20}$$

A solution to this equation for harmonic, spherical diverging waves is

$$\overline{p} = \frac{\overline{A}}{r}\exp[i(\omega t - kr)]. \tag{2.21}$$

The particle velocity for spherical waves is given by:

$$\underline{u} = \left(1 - \frac{i}{kr}\right)\frac{\overline{p}}{\rho_o c}\hat{r}. \tag{2.22}$$

Notice that the particle speed and pressure are out of phase in this case, whereas they were in phase for plane waves. Note, too, that for large values of kr the particle velocity tends to the plane wave result that:

$$\lim_{r\to\infty}\underline{u} = \frac{\overline{p}}{\rho_o c}\hat{r}. \tag{2.23}$$

2.3 Acoustic intensity

The intensity, I, of an acoustic wave is the average rate of flow of energy through a unit area normal to the propagation direction. Its units are Wm^{-2}.

It can be calculated from the pressure and the particle velocity in the following way.

$$I =< pu >_T= \frac{1}{T}\int_0^T pu\,dt, \tag{2.24}$$

where T is the period of a complete cycle.

In the case we have been considering of a harmonic plane wave travelling in the positive x direction, we find that the intensity is

$$I = \frac{p^2}{2\rho_o c}. \tag{2.25}$$

2.4 Specific acoustic impedance

This is defined as the ratio of the acoustic pressure to the particle speed:

$$\overline{z} = \frac{\overline{p}}{\overline{u}}, \tag{2.26}$$

where $\overline{z}$ is the specific acoustic impedance. Note that the specific acoustic impedance may be complex, and that in this form the equation is completely general. However, for plane waves:

$$\overline{z} = \pm\rho_o c, \tag{2.27}$$

the sign of $\overline{z}$ depending on the direction of propagation.

$\rho_o c$ is called the characteristic acoustic impedance of the medium through which the acoustic wave is propagating. The units of specific acoustic impedance are Pascals seconds per metre, $Pa\,sm^{-1}$, often called the rayl after Lord Rayleigh, the 19th century physicist.

The concept of impedance has been borrowed from electricity and electromagnetism. It is often useful to make analogies between acoustic field quantities and field quantities in electricity and electromagnetism, and also between acoustical and mechanical vibrations. Such analogies are possible because the linear differential equations which govern the mechanical and acoustic cases have the same form as those governing electromagnetic disturbances. The most commonly used analogies are those in which electrical voltage corresponds to mechanical force or to sound pressure and electrical current corresponds to velocity. Thus, it is clear that the relationship $\overline{z} = \overline{p}/\overline{u}$ has the same form as Ohm's Law.

We can make similar analogies between acoustic waves travelling along a tube, for example, and electrical transmission lines. In the case of a closed tube (an acoustically hard termination) the acoustic velocity (current) at the termination is zero, corresponding to an open electrical circuit. For an open tube (acoustically soft), where the sound pressure (voltage) at the end of the tube is zero, we have a short circuit.

For spherical waves the specific acoustic impedance is:

$$\overline{z} = \rho_o c \frac{kr}{\sqrt{1 + (kr)^2}} \exp(i\theta), \tag{2.28}$$

$$\overline{z} = \rho_o c \cos(\theta) \exp(i\theta), \tag{2.29}$$

where $\cot(\theta) = kr$.

Note that $kr = 2\pi r/\lambda$, so that the phase angle is a function of the ratio of the distance to the acoustic wavelength. The phase difference between pressure and velocity is large at small distances and small at large distances.

The magnitude of the specific acoustic impedance for spherical waves is:

$$z = \frac{P}{U} = \rho_o c \cos\theta, \tag{2.30}$$

where P is the acoustic pressure amplitude, and U is the particle speed amplitude.

Question 2 Show that for a harmonic spherical wave the intensity, I is given by:

$$I = \frac{P^2}{2\rho_o c} \tag{2.31}$$

and is identical to the intensity for plane waves.

2.5 Wave speeds

In this section we will examine some results for the speed of the sound wave in different circumstances. It is important to recognise that the wave speed and the acoustic particle speed are different physical quantities.

2.5.1 Sound speeds in solids

To calculate the speeds of different types of wave in solid materials we need to know the various elastic constants for homogeneous isotropic solids.

The two fundamental constants which relate changes in the shape of solids to applied stress are the bulk modulus, K, and the shear modulus, G. In addition, we shall use Young's modulus or longitudinal elasticity, E, the compressibility κ (given by the reciprocal of K), and the longitudinal modulus. Finally, we shall also need Poisson's ratio, ν, which is the lateral contraction per unit breadth divided by the longitudinal extension per unit length under an applied longitudinal stress. We therefore have the following relationships:

$$p = -K\frac{\Delta V}{V}, \tag{2.32}$$

where p is pressure and V is volume.

$$G = \frac{E}{2(1+\nu)}; \tag{2.33}$$

$$K = \frac{E}{3(1-2\nu)} = \frac{1}{3}\frac{EG}{(3G-E)}. \tag{2.34}$$

Note that Poisson's ratio is usually positive and typically has values between 0 and 0.5. The units of E, G and K are Pascals. Values are usually quoted in GPa.

In isotropic solids shear and longitudinal waves can be propagated, whereas in fluids (and this is often taken to include soft biological tissues) only longitudinal waves propagate, as shear waves are rapidly attenuated. The velocity of shear waves in an extensive medium (and also of torsional waves in thin cylindrical bars) is:

$$c_S = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2\rho(1+\nu)}}. \tag{2.35}$$

The speed of longitudinal or irrotational waves in an extensive medium is:

$$c_L = \sqrt{\frac{(K + \frac{4}{3}G)}{\rho}} = \sqrt{\frac{E(1-\nu)}{\rho(1-2\nu)(1+\nu)}}. \tag{2.36}$$

In straight uniform bars and in tubes which are thin compared with the wavelength of sound (and we can ignore Poisson's ratio) the speed of longitudinal waves is:

$$c_E = \sqrt{\frac{E}{\rho}}. \tag{2.37}$$

Waves which propagate along the surface of an extensive solid are known as Rayleigh waves and they travel with a speed of $c_{SR} = ac_s$ where a is the least positive root of the equation:

$$\frac{a^6}{8(1-a^2)} + a^2 = \frac{1}{1-\nu} \tag{2.38}$$

In plates the propagating modes are dispersive (sound speed varies with frequency). Anisotropic solids may have up to 21 independent elastic constants, and for any given direction there may exist three distinct plane wave velocities. These topics should be covered in the third year "Waves in Solids" course, and we won't deal with them further in this course.

2.5.2 Sound speeds in gases

In gases at moderate pressures the speed of sound is:

$$c = \sqrt{\frac{\gamma P}{\rho}}, \tag{2.39}$$

or alternatively

$$c = \sqrt{\frac{\gamma RT}{M}}, \tag{2.40}$$

where P is the ambient gas pressure, γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume, R is the gas constant, T is the absolute temperature, and M the molecular weight. Note that to first order the sound speed is proportional to the square root of the absolute temperature. This is not true at higher pressures.

The speed of sound waves which are bounded by walls or tubes is less than the free-space value.

2.5.3 Sound speeds in liquids

The speed of sound in fluids is given by:

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_{adiabatic}} \tag{2.41}$$

where P is the instantaneous pressure at a point and the partial derivative is evaluated at equilibrium conditions of pressure and density. The speed of sound in fluids depends on the thermodynamic variables of temperature, pressure and density. For perfect gases we have seen that the equation can be simplified to the forms given above.

If B is the adiabatic bulk modulus for fluids, then the speed of sound is given by:

$$c = \sqrt{\frac{B}{\rho_o}}. \tag{2.42}$$

For data on the speed of sound in pure water as a function of temperature see Del Grosso V.A. and Mader C.W. (1972) Speed of sound in pure water, J. Acoust. Soc. Am, 52, pp 1442-1446. At 20^{o}C the speed is 1482 ms^{-1}.

2.6 Reflection and transmission of sound waves at boundaries

We will concern ourselves only with plane waves and consider some of the more simple cases of reflection and transmission. In practice, these will be sufficient for us to understand most of the experimental situations we shall encounter. Firstly, remember that the characteristic acoustic impedance of a material is given by the product of its density and the speed of compressional waves in the material. We shall use Z_1 as the acoustic impedance of material 1 and Z_2 for the acoustic impedance of material 2 and we shall assume that the sound waves are travelling from material 1 into material 2. At an interface between two media some part of the sound energy will be transmitted and some reflected and our aim is to be able to calculate the ratio of transmitted or reflected pressure (or intensity) to the incident pressure (or intensity).

2.6.1 Interface between two fluid media

Wave incident normally at the boundary

We shall not derive the result, except to note that the derivation relies on the assumption that there is continuity of acoustic pressure, and continuity of acoustic particle velocity normal to the boundary. In addition, the frequency of the transmitted wave is the same as that of the incident wave although the phase speeds, c_1 and c_2 are different, so that the wave numbers $k_1 = \omega/c_1$ and $k_2 = \omega/c_2$ are different in the two media. Finally, we shall assume that there is no attenuation of the wave in the media at each side of the boundary.

The pressure amplitude reflection coefficient is given by:

$$\frac{p_r}{p_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{2.43}$$

where p_r is the reflected pressure and p_i is the incident pressure. Here we are using the notation employed by Dr. Chivers in Chapter 2 of "Output measurements for medical ultrasound", editor Dr R.C. Preston, (1991), Springer-Verlag.

The pressure transmission coefficient is:

$$\frac{p_t}{p_i} = \frac{2Z_2}{Z_1 + Z_2} \tag{2.44}$$

For reflection of intensities we have:

$$\frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \tag{2.45}$$

and for the transmission of intensities:

$$\frac{I_t}{I_i} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2} \tag{2.46}$$

Two fluid media - oblique incidence

If waves are incident on the boundary at an angle to the normal to the boundary of θ_i, there will be a wave which is reflected at the angle $\theta_r = \theta_i$. Snell's laws govern the transmitted wave, which will be refracted according to the relationship:

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i, \tag{2.47}$$

where θ_i and θ_t are the angles of the incident and transmitted waves relative to the normal to the boundary, and c_1 and c_2 are the sound speeds in the media on either side of the boundary with the assumption that the sound wave is travelling from medium 1 to medium 2.

In this case the pressure reflection coefficient is given by:

$$\frac{p_r}{p_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}, \tag{2.48}$$

and the pressure transmission coefficient is:

$$\frac{p_t}{p_i} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}. \tag{2.49}$$

For reflection and transmission of intensities the respective results are:

Reflection:

$$\frac{I_r}{I_i} = \left[\frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t}\right]^2; \tag{2.50}$$

Transmission:

$$\frac{I_t}{I_i} = \frac{4Z_2Z_1\cos\theta_i\cos\theta_t}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2}. \tag{2.51}$$

Note that if the transmitted wave angle θ_t is 90^o then Snell's Law gives:

$$\sin\theta_i = \sin\theta_c = \frac{c_1}{c_2} \tag{2.52}$$

The angle θ_c is called the critical angle since no acoustic energy is transmitted for angles of incidence greater than this.

In addition, if the angle of incidence θ_i approaches 90^o then $\cos\theta_i$ tends to zero so that the intensity reflection coefficient becomes:

$$\frac{I_r}{I_i} = \left[-\frac{Z_1\cos\theta_t}{Z_1\cos\theta_t}\right]^2 = 1, \tag{2.53}$$

so that once again no acoustic energy is transmitted - this condition is known as grazing incidence.

2.6.2 Reflection and transmission at the surface of solids

In general reflection and transmission at solid surfaces is more complicated than that between two fluids. In the case of a boundary between a fluid and a solid, longitudinal waves in the fluid may be converted in part to transverse waves in the solid, so that both longitudinal and shear waves may propagate in the solid. The term "mode conversion" is used to describe such conversions. For normal incidence a solid can be treated as a fluid because transverse waves will not be formed in the solid.

For the simplest case of oblique incidence in solids, consider a plane acoustic wave travelling through a fluid which meets the surface of a normally reacting solid. The normal specific acoustic impedance, z_n, is defined as the ratio of the acoustic pressure acting on the surface of the solid to the particle velocity of the fluid normal to the solid surface. The impedance may be complex, so that $z_n = r_n + ix_n$, where r_n and x_n are the real (resisitive) and imaginary (reactive) parts of z_n. In this case the pressure reflection coefficient is:

$$\frac{p_r}{p_i} = \frac{(r_n \cos\theta_i - \rho_1 c_1) + ix_n \cos\theta_i}{(r_n \cos\theta_i + \rho_1 c_1) + ix_n \cos\theta_i}. \tag{2.54}$$

The sound intensity reflection coefficient is:

$$\frac{I_r}{I_t} = \frac{(r_n \cos\theta_i - \rho_1 c_1)^2 + x_n^2 \cos^2\theta_i}{(r_n \cos\theta_i + \rho_1 c_1)^2 + x_n^2 \cos^2\theta_i}; \tag{2.55}$$

and the sound intensity transmission coefficient is:

$$\frac{I_t}{I_i} = \frac{4\rho_1 c_1 r_n \cos\theta_i}{(r_n \cos\theta_i + \rho_1 c_1)^2 + x_n^2 \cos^2\theta_i}. \tag{2.56}$$

2.6.3 Transmission through three media

It is of importance for practical applications of ultrasound, particularly in non-destructive testing, medicine, and also for the design of transducers, to examine what occurs when sound travels through three joining materials. We shall consider only plane waves which are normally incident on the surface of each material and we shall assume that all three media are homogeneous and isotropic. Once again, the direction of travel of the wave is from the first medium. The third medium extends to infinity, but the middle material is of thickness L. Because we have specified normal incidence we can apply the results to both fluids and solids. If the impedances of the three materials (the product of density and the speed of compressional waves) are Z_1, Z_2 and Z_3, then the sound pressure transmission coefficient from medium 1 to medium 3 is:

$$\frac{p_t}{p_i} = \frac{2Z_2 Z_1}{Z_2(Z_3 + Z_1)\cos k_2 L + i(Z_2^2 + Z_3 Z_1)\sin k_2 L}. \tag{2.57}$$

The sound intensity transmission coefficient is:

$$\frac{I_t}{I_i} = \frac{4Z_3 Z_1}{(Z_3 + Z_1)^2 \cos^2 k_2 L + (Z_2 + Z_3 Z_1/Z_2)^2 \sin^2 k_2 L}. \tag{2.58}$$

We can consider some special cases. Firstly, if medium 1 and 3 are the same, $Z_1 = Z_3$, so that:

$$\frac{I_t}{I_i} = \frac{4}{4\cos^2 k_2L + (Z_2/Z_1 + Z_1/Z_2)^2 \sin^2 k_2L}. \tag{2.59}$$

Next, for transmission from a rarefied medium through a dense medium into the same rarefied medium, $Z_2 \gg Z_1$, we have:

$$\frac{I_t}{I_i} = \frac{4}{4\cos^2 k_2L + (Z_2/Z_1)^2 \sin^2 k_2L}. \tag{2.60}$$

Suppose that the rarefied medium is air (for example, sound travelling from one room through a solid wall into another room), then

$$\frac{Z_2 \sin k_2L}{Z_1} \gg 2\cos k_2L. \tag{2.61}$$

Except for very large L and for high frequency sound (large k), $k_2L \ll 1$ and $\sin k_2L \simeq k_2L$, we then obtain our simplest result:

$$\frac{I_t}{I_i} = \frac{4Z_1^2}{Z_2^2 k_2^2 L^2} \tag{2.62}$$

2.6.4 Impedance matching: an example

We require maximum transmission of plane acoustic waves from water into steel. What is the optimum characteristic impedance of the material to be placed between the water and the steel?

The sound energy transmission coefficient is:

$$\frac{I_t}{I_i} = \frac{4Z_3Z_1}{(Z_3 + Z_1)^2 \cos^2 k_2L + (Z_2 + Z_3Z_1/Z_2)^2 \sin^2 k_2L}. \tag{2.63}$$

We note that if $k_2L = (2n-1)\pi/2$ where $n = 1, 2, \ldots$, then $\sin k_2L = 1$ and $\cos k_2L = 0$, so that we obtain:

$$\frac{I_t}{I_i} = \frac{4Z_3Z_1}{(Z_2 + Z_3Z_1/Z_2)^2}. \tag{2.64}$$

For maximum transmission we require that $I_t/I_i = 1$, so that solving for Z_2 gives:

$$Z_2 = \sqrt{Z_1 Z_3}. \tag{2.65}$$

But we also specified that $k_2 L = (2n-1)\pi/2$. This means that 100% transmission only occurs for a specific frequencies and thicknesses of intermediate layer. Remembering that $k = 2\pi/\lambda$ and $c = f\lambda$, and for $n = 1$, we find that the thickness of the layer must be:

$$L = \frac{\lambda}{4} \tag{2.66}$$

and the speed of sound is:

$$c_2 = 4Lf \tag{2.67}$$

where c_2 is the speed of longitudinal waves in the matching layer.

The density of the required material is $\rho_2 = Z_2/c_2$.

Question 3 Look up densities and sound speeds for water, steel and some other materials in Kaye and Laby or an alternative source and try to choose possible materials for a matching layer at 500 kHz. How thick should the matching layer be?

2.7 Mode conversion - practical consequences in non-destructive testing

We noted that solids can support the propagation of shear waves, and at oblique incidence at a solid surface a compressional wave may be converted into a shear wave. At a solid/solid interface there may be both reflected and transmitted shear waves, as well as reflected and transmitted compressional waves. At the solid interface the directions of propagation of the two types of wave are governed by Snell's laws, as we have already seen. Consider a plane compressional wave meeting a solid surface at an angle θ_i to the surface normal. The subscripts 1 and 2 indicate the two media, where the wave travels from 1 to 2. Let θ_L be the angle to the normal of the compressional wave in the solid and let θ_S be the angle to the normal of the shear wave in the solid. Let c_{L1} be the speed of longitudinal or compressional waves in material 1 and c_{L2} be the compression wave speed for material 2. In addition, the speed of shear waves in material two is c_{S2}. Then in material 2 we have for the refracted compression wave component:

$$\frac{\sin\theta_i}{\sin\theta_L} = \frac{c_{L1}}{c_{L2}}, \tag{2.68}$$

and for the refracted shear wave component:

$$\frac{\sin\theta_i}{\sin\theta_S} = \frac{c_{L1}}{c_{S2}} \tag{2.69}$$

During ultrasonic testing of materials two waves travelling at different speeds may give confusing results, so that it becomes necessary to adjust the angle of incidence to allow only one beam to propagate. For the case in which sound travels faster in material 2 than in material 1, which is generally the case for inspection of metallic components where the transducer crystal may be mounted in an angled perspex block (we shall consider transducer construction in greater detail in subsequent lectures), we can choose to launch waves into the material at an angle greater than the critical angle for compression waves (which travel faster than shear waves) so that only shear waves propagate. The critical angle for compression waves is given by $\sin^{-1}(c_{L1}/c_{L2})$, so that for angles greater than this, but less than the critical angle for shear waves, only shear waves will be propagated.

It is possible to use Rayleigh or surface waves in the inspection of some components and in this case the critical angle for the generation of Rayleigh waves is given by $\sin^{-1}(c_{L1}/c_{S2})$.

For a perspex/steel interface the critical angle for compression waves is 27.5^o and the critical angle for Rayleigh waves is 57^o.

2.8 Attenuation

So far we have ignored losses due to propagation through materials. An acoustic signal may be attenuated for three main reasons. Sound energy may be absorbed by the medium; the profile of the propagating beam may be changed through diffraction or beam spreading; and finally the acoustic wave may be scattered by inhomogeneities in the material. We have already seen that at interfaces between materials, only a part of the acoustic energy may be transmitted.

In this course we don't have the time to investigate all the mechanisms by which acoustic energy is absorbed by gases, liquids and solids. We will, however, briefly examine losses due to scattering in subsequent lectures, as this is important in the ultrasonic characterisation of inhomogeneous materials.

2.8.1 Attenuation coefficient

We can define a linear, plane wave attenuation coefficient for materials in the following way. If $p(0)$ is the acoustic pressure at $x = 0$, where x is a position co-ordinate, and $p(x)$ is the pressure at point x, then:

$$p(x) = p(0)\exp(-\mu x) \tag{2.70}$$

where μ is the attenuation coefficient in units of nepers per metre.

Attenuation is often described in units of decibels per metre so that:

$$\alpha = 20(\log_{10} e)\mu \tag{2.71}$$

where α is the attenuation coefficient in decibels per metre.

For plane waves in a homogeneous medium where there is no scattering then the attenuation coefficient may be described as the absorption coefficient. The intensity attenuation coefficient for plane waves in homogeneous media is twice the pressure attenuation coefficient.

2.8.2 Attenuation as a function of frequency

In general, absorption is a function of frequency, as are losses due to scattering. For homogeneous solids absorption varies as the first power of the frequency. For soft biological tissues absorption typically varies as $f^{1.1}$ to $f^{1.3}$. For water the absorption coefficient is proportional to the square of the frequency. In the case of castor oil, a viscoelastic fluid, absorption is proportional to $f^{5/3}$. Absorption is often stated in terms of losses of dB/cm/MHz. Absorption is also a function of temperature, and experimental determinations of attenuation losses should quote both the measurement frequency and temperature, as should measurements of sound speeds.

Chapter 3

Piezoelectricity, transducers and hydrophones

3.1 Generating ultrasound

In order to generate ultrasonic waves we need a transducer of some kind. Early experimenters used whistles, sirens and tuning forks, and also liquid-driven transducers such as hydrodynamic oscillators. These devices operate at relatively low ultrasonic frequencies.

One electromechanical means of generating ultrasound relies on the magnetostrictive effect. When a rod of magnetisable material is exposed to a magnetic field which varies in magnitude the rod changes in length. An alternating current passing through a coil surrounding the rod causes it to vibrate longitudinally. These forced vibrations increase greatly in amplitude if the frequency of the applied current coincides with one of the normal longitudinal modes of vibration of the rod. Such transducers are made from alloys of iron, nickel and cobalt. They are mechanically rugged devices and are capable of producing large acoustic powers with reasonable efficiency, but are limited to frequencies of about 100 kHz.

Finally, ultrasound may be generated electromagnetically using what is effectively a further development of conventional loudspeakers used at audio frequencies.

3.2 Piezoelectricity

A useful additional reference to look at in conjunction with this section can be found at `http://gummo.stanford.edu/DD/Courses/me220/lectures/lect06/lect_6.html`

By far the most common means of generating ultrasonic signals relies on the piezoelectric effect.

Piezoelectricity is the property possessed by some materials of becoming electrically charged when subjected to a mechanical stress. Such materials also exhibit the converse effect, that is, the occurrence of mechanical deformation on the application of an electric field.

The piezoelectric effect was first observed in naturally occurring single crystal compounds such as quartz and Rochelle salt. The exhibition of piezoelectricity by such compounds is due to the lack of a centre of symmetry in the unit cell and consequently distortion of the unit cells produces electric dipoles.

3.2.1 Piezoelectric ceramics

Certain compounds can be made piezoelectric by the application of a high electric field (polarisation). These are termed ferroelectric materials. Examples of such materials are barium titanate and lead titanate which can be produced as single crystals or as polycrystalline aggregates by the ceramic process.

Before polarisation the dipoles are randomly oriented and the material is isotropic. The polarisation process involves the application of an electric field across the ceramic, usually at elevated temperatures, causing switching or realignment of the dipoles. A graph of polarisation versus applied field yields a closed curve analogous to the magnetic hysteresis loop. After removal of the electric field there is a remanent polarisation which is responsible for the piezoelectric properties. The resulting ceramic is now anisotropic. The ceramic can now be returned to its unpolarised isotropic condition by raising its temperature above the Curie point or by mechanical or electrical overstressing.

The alignment of dipoles in a polycrystalline ferroelectric is not as great as in a single crystal but the convenience with which polycrystalline materials can be produced compared with single crystals results in their extensive use in many applications.

Because piezoelectric ceramics are anisotropic and the direction of polarisation may be freely chosen, a method of identifying the axes of a component and the direction of polarisation is necessary in order to specify its parameters.

Piezoelectric ceramics are used for transducers employed in ultrasonic cleaning, welding and drilling, non-destructive testing, medical diagnosis and therapy, underwater transducers including echo sounders, sonobuoys, sonar, transponders, hydrophones, accelerometers and strain gauges. They cover a wide frequency range from a few kilohertz to tens of MHz.

For further information on the topics mentioned above, refer to the piezoelectric ceramics data tables produced by the company Morgan Matroc Ltd, which can be found at:

`http://www.matroc.com/mmecframe.html` (Fom here you need to do a bit of navigating through the frames: Choose "Products" in the left-hand frame, then scroll down to "Electroceramics Division, Ohio", then, finally, "Online Catalog", followed by "PZT Specifications", "PZT Ceramic Materials" or "PZT Properties")

Exercise: These notes have summarised the main topics you need to understand about piezoelectric ceramics. Read through the Morgan Matroc text on the accompanying pages and make your own additional notes.

From a study of the data book you will have realised that there are three important material properties of piezoelectric ceramics which designers need to take into account when they choose materials for a particular transducer application. These are:

- the transmitting constant, d, which is the strain developed for a unit applied field (metres per volt);
- the receiving constant, g, which is the open circuit field per unit applied stress (volts per newton).
- the electromechanical coupling coefficient, k, which is the square root of the ratio of the stored mechanical energy to the applied electrical energy.

Remember that transducers are used both as transmitters and receivers, and that for pulse-echo techniques the same transducer is used for sending and receiving. For transmitting ultrasound a high value of d is an advantage. In reception a high g value is of benefit as this will ensure good sensitivity to small echoes. A useful figure of merit to use as an indicator here is the product dg.

3.2.2 Piezoelectric polymers

A material which is now becoming much more common for both transducers and hydrophones is the piezoelectric polymer polyvinylidene fluoride (often referred to as PVDF). The value of d for PVDF is up to about 20 times less than that of lead zirconate titanate ceramics, but its receiving constant g can be about seven times higher, making it particularly useful for hydrophones. The characteristic acoustic impedance of PVDF is also closer to water (and biological tissue) than lead zirconate titanate, so that energy is more efficiently transferred between the transducer and water (or tissue).

We can regard a piezoelectric or ferroelectric material as a capacitor with a capacitance, C, given by:

$$C = \frac{\epsilon A}{t} = \frac{Q}{V}, \tag{3.1}$$

where ϵ is the absolute permittivity of the dielectric, A is the area of the electroded surfaces, t is the thickness of the dielectric, Q is charge (coulombs) and V is the potential across the dielectric (volts). In the case of PVDF ϵ is $0.98.10^{-11}\ Fm^{-1}$, whereas for PZT5a, for example, $\epsilon = 1500.10^{-11}\ Fm^{-1}$. Thus it is much easier to transfer electrical energy into PZT5A than it is into PVDF. PVDF transmitters are not as efficient as ceramic transducers. However, PVDF devices are good broadband receivers (membrane hydrophones and PVDF needle hydrophones).

For more information on PVDF, consult the reference
`http://www.ndt.net/article/yosi/yosi.htm#2`.

3.3 Transducers

We shall consider three types of transducer used for generating ultrasonic signals: the single element plane piston, the single element focused bowl transducer, and the multi-element array transducer. However, for the purposes of understanding the acoustic fields radiated by transducers we shall concentrate on the single element plane piston device.

3.3.1 Plane piston transducer

Construction

A piezoelectric ceramic transducer is constructed so that the element resonates mechanically in a thickness mode at a particular frequency, that is, its thickness is equivalent to half a wavelength. Thus, if we know the speed of sound in the ceramic and its thickness, we can predict the resonant frequency of the transducer.

For example, the speed of sound in PZT4 is 4,000 ms^{-1}. Suppose that the active element of a transducer is 2 mm thick. $\lambda/2 = 2\,mm$ and using $c = f\lambda$, we find that the resonant frequency is 1 MHz. For this material a 10 MHz transducer would be 0.2 mm thick. It is difficult to make ceramic transducer elements suitable for high frequency applications (10 MHz or greater) which are larger than about 19 mm in diameter. For PZT5A the speed of sound is 3780 ms^{-1}, so that in thickness mode, the resonant frequency for a 2 mm thick element is 1.06 MHz.

The choice of backing material used behind the ceramic element can affect the way in which the transducer behaves. If one requires continuous waves at a particular frequency, no backing material is used (the transducer is described as being "air-backed"). For short pulses an attenuating backing material is used. Even if a very short drive pulse is used, the transducer element tends to "ring" at the resonant frequency. The "ringing" can be reduced electrically or mechanically. If mechanical methods are chosen, the mass of the element is increased by adding a backing material (typically epoxy resin with added tungsten powder). The backing not only shortens the pulse but also tends to lower the sensitivity and to reduce the operating frequency. However, the shorter pulse means that the transducer has a broader bandwidth.

A protective layer is mounted on the element's front face. We have already seen that it may be possible to maximise transmission and reception through a middle layer of material at a particular frequency through the correct choice of acoustic impedance and thickness. If one knows how the transducer is to be used (for example, immersed in water, or for non-destructive testing of metals), a suitable $\lambda/4$ matching layer can be selected. If the transducer is being used in pulsed mode, then it is only possible to optimise for one particular frequency within the transducer's bandwidth.

Field radiated by a plane piston transducer

Suppose we have a circular plane piston mounted flush with the surface of an infinite baffle, and that the surface of the transducer vibrates with simple harmonic motion. At each point on the surface the vibration is of the same amplitude and in phase with all other points. Suppose also that we divide the surface into infinitesimally small elements. Then each element acts like a baffled simple source.

We also assume that the piston is radiating into a homogeneous, loss-less medium.

If r is the distance from some arbitrary point in the field to the surface element dA, then the pressure at the field point can be obtained by integrating over the surface.

The pressure at a point z is given by:

$$\overline{p}(z,t) = i\frac{\rho_o c U_o k}{2\pi}\int_S \frac{\exp i(\omega t - kr)}{r} dA, \tag{3.2}$$

where $U_o \exp(i\omega t)$ is the speed at which the piston surface moves normal to the baffle.

We can take the time dependence outside the integral and can ignore the constants by writing:

$$\overline{p}(z,t) \propto \int_s \frac{\exp(-ikr)}{r} dA. \tag{3.3}$$

Note the $1/r$ dependence we observed for spherical waves radiated from a point source.

Calculation of this integral for an arbitrary position in the field involves considerable difficulty and in most cases it is not possible to derive closed form solutions. The only cases in which such solutions can be obtained are for points along the axis of the transducer beam and at large distances from the transducer, in what is referred to as the "far field".

We shall consider points on the acoustic axis and the distance from the piston surface along the axis is denoted by z. The radius of the surface is R, and points along the radius will be denoted by a. If angles are denoted by ϕ, then the area of a surface element dA is given by $dA = a\, d\phi\, da$.

The distance from an element dA to the point z is given by:

$$r^2 = a^2 + z^2, \tag{3.4}$$

and by differentiating with respect to da we find that:

$$r\, dr = a\, da. \tag{3.5}$$

Considering just the z-dependence of $\overline{p}$, the integral we now have to solve is:

$$\overline{p}(z) \propto \int \frac{\exp(-ikr)}{r} a\, d\phi\, da. \tag{3.6}$$

$$\overline{p}(z) \propto \int_0^{2\pi} d\phi \int_0^R \frac{\exp(-ikr)}{r} a\,da \tag{3.7}$$

Integrating over the range of $d\phi$ and changing $a\,da$ to $r\,dr$ (and remembering to change the limits of the integral when we do so), we get:

$$\overline{p}(z) \propto \int_z^{(R^2+z^2)^{1/2}} \frac{\exp(-ikr)}{r} r\,dr \tag{3.8}$$

$$\overline{p}(z) \propto \exp\left[-ik(R^2+z^2)^{1/2}\right] - \exp(-ikz) \tag{3.9}$$

This result shows that the sound field on the axis of a circular piston can be interpreted as the superposition of a plane wave $\exp(-ikz)$ propagating perpendicular to the plane and a diffracted wave of equal amplitude and opposite phase arising from the edge of the piston, $\exp[-ik(R^2+z^2)^{1/2}]$.

Using the identity that:

$$\exp(-i\alpha) - \exp(-i\beta) = -2i\sin(\frac{\alpha-\beta}{2})\exp[-i(\alpha+\beta)/2], \tag{3.10}$$

our final form for $\overline{p}$ is:

$$\overline{p}(z) \propto \sin\left(\frac{k}{2}[(R^2+z^2)^{1/2}-z]\right)\exp\left(i\omega t - i\frac{k}{2}[(R^2+z^2)^{1/2}+z]\right). \tag{3.11}$$

The sound pressure on the axis goes to zero every time the sine term goes to zero, so that the pressure is zero when:

$$\frac{k}{2}[(R^2+z_n^2)^{1/2}-z_n] = n\pi, \qquad n = 1, 2, 3, \ldots \tag{3.12}$$

Thus there are pressure nulls when:

$$z_n = \frac{(R/\lambda)^2 - n^2}{2n/\lambda}, \qquad n = 1, 2, 3, \ldots \tag{3.13}$$

When the radius R is smaller than the wavelength λ the last equation cannot be satisfied for any value of n, so that there are no sound pressure nulls on the axis. For larger diameters, $R > \lambda$, there are nulls, but only a finite number are possible because z_n becomes negative at a particular value of n and beyond. (Note that as n increases z_n gets smaller: increasing n means movement towards the transducer.) The sound pressure reaches maxima of constant amplitude between each two nulls.

The positions of axial maxima are given by:

$$z_{max} = \frac{4R^2 - \lambda^2(2n+1)^2}{4\lambda(2n+1)}, \qquad n = 0, 1, 2, 3, \ldots \tag{3.14}$$

The location of the last axial maximum (furthest from the transducer) is:

$$z_o = \frac{4R^2 - \lambda^2}{4\lambda} \tag{3.15}$$

Note that if $\lambda \ll R$ then the last axial maximum is at $z_o \simeq R^2/\lambda$.

Far field of a plane piston source

In the far field of a plane piston source, the pressure is given by:

$$\overline{p}(r,\theta,t) = i\frac{\rho_o c}{2} U_o \frac{R}{r} kR \exp(i[\omega t - kr]) \left[\frac{2J_1(kR\sin\theta)}{kR\sin\theta}\right], \tag{3.16}$$

where r is the distance from the centre of the circular disc to the field point, R is the radius of the transducer, θ is the angle between the normal to the disc and the field point r and J_1 is the Bessel function of the first kind of order one. It is important to recognise two main features of this result. The far field pattern of a plane piston source is a spherical wave which is modified by a directivity function.

$$\overline{p}(r,\theta,t) \propto \underbrace{\frac{\exp[i(\omega t - kr)]}{r}}_{spherical\ wave} \underbrace{\frac{2J_1(kR\sin\theta)}{kR\sin\theta}}_{directivity\ function} \tag{3.17}$$

Exercise

Write computer programs which:

1. Calculate the axial pressure distribution of a plane piston source for various values of kR;

2. Calculate the far field distribution of a plane piston source as a function of angle, θ, for a particular r value, and a range of kR values.

What do you observe in each case as kR gets larger?

If you use computing algebra packages such as Maple or Mathematica you won't have to program the Bessel function routine yourself. Otherwise, a way of calculating Bessel functions is given in the Book "Numerical Recipes".

Remember that all the results we have derived so far have been for ideally-behaved devices radiating continuous waves at a single frequency. In practice we are unlikely to encounter such a situation, and in many cases the only way in which to understand the field radiated by a particular device is to measure it.

One method of accounting for departures from ideal behaviour is the concept of the "effective radius". It may be that the field pattern does not coincide with that of a plane piston of the geometrical radius in question. However, it could be that the use of a different radial distance may lead to better predictions of the field. Is so, then this alternative distance is called the "effective radius". This approach still relies on the assumption that the device is a perfect plane piston so that it still includes the assumptions of circular symmetry and continuous waves of a single frequency. The size of the "effective radius" may be derived from measurements of the beam profile at a known position in the field and applying the theory of plane piston sources we have just reviewed.

If a plane piston is used in pulsed mode, the fields it generates will in general be simpler and have less structure than continuous wave fields. This is because the pulse exists for only a short period of time, so that there may be insufficient time for complicated interference patterns to build up. The shorter the pulse, the simpler the field structure is likely to be. But remember that the shorter the pulse, the greater the range of frequencies it contains.

3.3.2 Focused single element transducers

In order to obtain narrower beamwidths than are possible with a plane transducer or to deliver increased acoustic energy to a particular point, we need to use focused transducers. The wavefronts have to be formed into a concave shape so that they converge to a focus. Between the transducer and the focal zone the waves converge. They are approximately plane within the focal zone and then they diverge or become convex beyond the focal zone. One method of focusing is to form the transducer into part of the surface of a sphere, by attaching a concave lens to its front face. Standard formulae from geometrical optics can be used to calculate the focal length.

Question: why do we need a concave lens for focusing in water or biological tissue?

In practice, the location of the point of maximum acoustic pressure is usually closer to the transducer than the geometrical focal point (the centre of curvature of the spherical bowl) but its actual position is a function of frequency.

3.3.3 Multi-element transducers

For some applications, and especially for diagnostic medical ultrasound scanners, transducers may consist of large numbers of small piezoelectric elements (often rectangular in shape) each of which has its own electrical excitation. By pulsing several elements at the same time or by varying the phasing of the pulse sequence, it is possible to focus the beam at different locations and to vary the width of the beam. In this way it is possible to scan a volume of tissue in order to build up an image. For example, a 5 MHz linear array transducer manufactured by Dynamic Imaging Ltd is about 10 cm long and 1 cm wide. It contains 79 separate piezoelectric elements, which are pulsed in various combinations to produce 160 different scan lines. Each frame of the diagnostic image contains information from the 160 lines. The scanner has three focal zones and four levels of magnification (image resolution) which are obtained by varying the pulsing pattern and excitation levels to different combinations of elements.

We will not be considering medical imaging in detail in this course, as this subject will be covered in much greater depth in the Level 2 Medical Acoustics module. However, you will find that many of the concepts employed in the pulse-echo investigation of materials and components in non-destructive testing using ultrasound, are also relevant to medical applications of ultrasound.

3.4 Hydrophones

In this section of the lectures we shall look at the use of miniature sensors for measuring acoustic fields at MHz frequencies. Such sensors are called hydrophones and they are frequently employed to characterise the acoustic fields radiated by transducers used for medical ultrasound and for non-destructive testing. These devices consist of thin plates or films of piezoelectric materials fabricated in the form of small sensing elements which convert the incident acoustic pressure waveform into an electrical signal. They combine high temporal and spatial resolution, are small and adaptable and can be combined easily with other instrumentation such as amplifiers and digital oscilloscopes.

The aim of any measurement of an acoustic field should be to obtain a measured waveform which is a faithful reproduction of the incident acoustic waveform. The extent to which this is achievable depends on the properties of the hydrophone. The properties of an "ideal" hydrophone can be summarised as:

- The device should be sufficiently sensitive to produce a reasonable signal level. They are not absolute measurement devices and their sensitivity, expressed in terms of volts per pascal, has to be determined by calibration in a known acoustic field.
- The voltage response of the hydrophone should be linear over the range of pressures likely to be encountered.
- The sensing or active element should be smaller or comparable in size to

the acoustic wavelength. For propagation in water in the range from 1 to 15 MHz, this implies diameters ranging from 1.5 mm to 0.1 mm.

- The hydrophone should show little variation in sensitivity with frequency over the bandwidth for which it is normally used for measurements.
- It should be robust and able to operate effectively under the acoustic pressure and power conditions it is likely to encounter.
- In order for it to be used as a reference working device, its sensitivity should be stable over a significant period of time.
- An ideal hydrophone should be omnidirectional, that is, its output for a particular incident pressure should not depend on the angle of incidence of the acoustic wave.
- It should not perturb the incident acoustic field.

In practice it is not possible to make hydrophones which possess all the above properties. The active elements of ultrasonic hydrophones are typically 0.5 mm to 1 mm in diameter, although smaller devices have been manufactured.

Precision Acoustics Ltd of Dorchester make a PVDF needle hydrophone of 75 microns diameter and have made devices of 40 microns diameter. Hewlett Packard also plan to market a hydrophone of 40 microns diameter. Devices with smaller elements tend to be less sensitive than those of larger diameter.

Diameters of 0.5 mm or so in MHz fields mean that hydrophones are strongly directional in their properties and they must be carefully aligned when one is making measurements. Sensitivities are typically of the order of 100 nanovolts per pascal and preamplifiers are often used to increase the signal level and act as an impedance buffer since the output impedance of the hydrophone is relatively high. Some needle probe hydrophones have the preamplifier incorporated into the body of the hydrophone.

There are three main types of miniature hydrophones: ceramic, PVDF needle probes and PVDF membrane hydrophones.

3.4.1 Ceramic hydrophones

Traditionally the active elements of hydrophones have been made from piezoelectric ceramics. The construction is usually in the form of a disc-shaped active element supported at the end of a pencil-like structure made of suitable absorbing material. Owing to the excitation of various resonance modes in the active element and reflections in the backing material, ceramic needle hydrophones often have unpredictable frequency and directional responses. They may also be unstable over time, so that for accurate work they should be calibrated frequently. However, they are robust devices and are particularly useful for narrow-band measurements where a flat frequency response is not as critical as it would be for pulsed fields.

3.4.2 PVDF needle-probe hydrophones

In this case the active element is PVDF and is supported on the end of a needle made from a thin-walled metal tube. The advantage of PVDF needle probes over ceramic needle probes is that the acoustic impedance of PVDF is more closely matched to that of water. However, they often show fluctuations in sensitivity in the region of 1 to 4 MHz.

3.4.3 PVDF membrane hydrophones

A membrane hydrophone consists of a thin sheet of unpoled PVDF film stretched over an annular ring (100 mm internal diameter) with gold/chromium electrodes vacuum-deposited onto the surfaces. These electrodes overlap only in a small central area, which is then poled to form the active element of the device. Typical element diameters are 1 mm and 0.5 mm, although 0.1 and 0.2 mm elements have been manufactured. There are two main types of device. The coplanar membrane consists of a single layer of PVDF. The bilaminar version consists of two layers of PVDF, with the active element and almost all of the live electrode sandwiched between two earthed plates.

Membranes have the advantage of causing minimal disturbance to the acoustic field since the acoustic field passes through the aperture of the ring in most cases. The frequency response of membrane hydrophones tends to be broadband and smooth. However the PVDF film will possess a thickness mode resonance at a frequency f such that:

$$f = \frac{c}{2d} \tag{3.18}$$

where d is the thickness of the film and c is the speed of sound in the PVDF (about 3000 ms^{-1}). For a device made from 25 micron thick film, this resonance occurs at around 40 MHz for a coplanar and 20 MHz for a bilaminar hydrophone, as it is double the thickness. Membranes as thin as 6 microns and 9 microns have been developed, and these have a correspondingly higher frequency resonance.

At low frequencies (1MHz) membrane hydrophones have large side lobes in their directional response, which is not the case for needle hydrophones. In addition, their effective radius at low frequencies is often larger than the geometrical radius, as the properties of PVDF film have the effect of increasing the effective receiving area.

Membrane hydrophones have stable sensitivities to within a few per cent over five years.

Exercise

Bearing in mind the comments made above about the three types of hydrophone, read through the paper by Preston et al. (1983), PVDF membrane hydrophones' performance properties and their relevance to the measurement of the acoustic output of medical ultrasonic equipment, J. Phys. E. Sci. Instrum., 16, pp 786-796, and make your own additional notes on the properties and performance of hydrophones.

Chapter 4

Using ultrasound to investigate the acoustic properties of materials

We have already seen that materials may absorb ultrasound, and that absorption is frequency dependent. We have also noted that ultrasound travels at different speeds in different materials. Measurements of these two parameters can be used to characterise both homogeneous and inhomogeneous materials. In the case of inhomogeneous materials we will be measuring some average value of absorption or speed, as these properties are likely to vary locally within the material. However, for such materials we may also be able to measure how they scatter ultrasound (that is, we may measure the scattering cross-section as a function of frequency and scattering angle).

4.1 Cautionary notes

In making such measurements of material properties it is common practice to assume that sound propagates as a plane wave. We have already seen that in many situations it is difficult to generate plane waves. In addition, there is an important property of piezoelectric receivers which can lead to errors when the plane wave model is applied. Piezoelectric receivers are sensitive to both the amplitude and phase of the acoustic signals they detect and their output is an average of the pressure fluctuations over their face. Suppose that one half of the receiver sees a wave of a particular amplitude and the other half sees a wave of the same amplitude but with a 180 degree phase reversal. The average complex pressure is in this case zero and the piezoelectric receiver would show no output voltage. Obviously this is an extreme case, but the effect is nevertheless real and should be borne in mind whenever one suspects that there are unwanted phase fluctuations across the face of a receiving transducer or hydrophone. In the scientific literature this phenomenon is often called the "phase cancellation artefact".

It is also important, even in the absence of phase fluctuations, to recognise that a receiver averages the acoustic field over its aperture. If there are pressure fluctuations, then the output will be an average of the incident pressure distribution. Measurements of the same field with receivers of different sizes may produce different results. This effect is known as "spatial averaging".

When one is using simple models of propagation based on the assumption of plane wave propagation, both "phase cancellation" and "spatial averaging" produce measurement "errors" which may require us to correct our raw data.

4.2 Measurement of speed

There are two main ways of measuring these properties: standing wave methods and pulse-echo methods. We are not going to carry out an exhaustive analysis of all the available techniques but will review some of the key methods.

4.2.1 Standing wave methods

Pierce's interferometer

It is possible to measure the speed of ultrasonic waves by forming stationary waves and, having determined the distance between successive nodes or antinodes, use the equation $c = f\lambda$ to determine the speed, given that the frequency of the transducer's electrical excitation is known. What is needed is a source of ultrasound and a reflector whose position can be varied. If the distance between the source and the reflector is an integral number of half wavelengths standing waves will be formed. One device which exploits this effect is Pierce's interferometer, which consists of a tube with a piezoelectric crystal oscillator at one end and a polished metal reflector attached to a micrometer screw thread at the other. The tube may be filled with liquid or gas. The crystal is also employed as a receiver, and in this case the voltage developed between a pair of electrodes on opposite surfaces of the piezoelectric transducer is amplified and processed electronically. If the anode current is measured by an ammeter it will be found that the amplitude of the current will be a periodic function of the reflector's position. If current is plotted against the distance the reflector is moved, then the difference in position between successive peaks represents one half wavelength of the ultrasound in the medium contained in the tube.

Optical methods in liquids

Ultrasonic waves can also be used to diffract light waves. We know that in an acoustic wave the pressure varies with position and time, and that there are corresponding space and time variations in the density of the medium through which the sound propagates. If the liquid is transparent there will be corresponding variations in the refractive index of light along the path of the ultrasonic wave. A standing wave pattern can thus be used as an optical diffraction grating. If a light source such as a laser beam is shone through the standing wave pattern there will be diffracted beams at angles θ from the undiffracted beam direction given by $\sin\theta = n\lambda_O/\lambda_S$, where λ_O is the optical wavelength, λ_S is the ultrasonic wavelength and n is an integer. Measurements of the optical diffraction pattern allow one to determine the ultrasonic wavelength, and, once again, if the frequency is known, the speed of sound can be determined.

Both techniques can be used to obtain accurate measurements of speed. They may also be employed to determine attenuation coefficients but this is more difficult and the results are less accurate.

4.2.2 Pulse-echo methods

Pulse-echo methods can be used to measure both the sound speed and attenuation of solid materials. Whereas, for standing wave methods, speed had to be calculated from the ultrasonic wavelength and frequency, in the case of pulse-echo the speed is determined from measurements of length and time. If the time taken for reflections from the back wall of a specimen to return to the transducer (which is used as both a transmitter and receiver) is measured using an oscilloscope, and the thickness of the sample is known, then the speed is simply distance divided by time, remembering, of course, that the signal has travelled through the sample and back, so that we must use twice the measured thickness in calculations. The standing wave methods used continuous signals but for pulse-echo we obviously have to use short pulses as the transmitted signal must only exist for a short period of time, so that the transducer can then be used to detect the returning echo.

A number of techniques can be used to make the time measurement, the simplest being for the operator to read the time directly from an oscilloscope screen. The most direct electronic methods rely on starting a timer on one signal, such as the trigger pulse to the transducer, and stopping it when the echo is received. However, there are various sources of error in such an approach. It is necessary to decide which feature of the echo signal is used to stop the timer, and if some threshold amplitude is chosen for this purpose, different shapes and amplitudes of echo pulse could lead to differences in the measured times. Noise may also cause echo pulse timing errors. For a non-rectified signal one may try to detect the time at which the echo signal goes from negative to positive, that is, to identify a zero-crossing point. Such a technique is also very sensitive to noise, so that in practice one may have to define a threshold amplitude, rather than a zero-crossing as the signal which switches off the timer. In addition, the timer clock will itself have a finite period, so that there will be an uncertainty in the timing of plus or minus one clock period. This kind of error can be reduced by averaging repetitive signals.

An alternative timing technique is the sing-around method, which uses the received echo pulse to trigger the next transmitted pulse. In this way the repetition frequency of the pulses becomes equal to the transit time for the specimen. The repetition frequency or period can be measured using standard laboratory counter timers. One of the limitations of such methods is that there is a systematic error which arises from the finite time it takes for the acoustic signal to travel through the transducer crystal and front face and also for the electronics to feed back the trigger pulse. The size of this systematic error may be quantified, however, by making measurements on samples of varying thicknesses, as the error will be independent of sample size.

Speeds can also be measured by determining the phase of the received signal but we won't consider such techniques in this course.

4.3 Measurement of attenuation

4.3.1 Pulse-echo methods

Using pulse-echo techniques to make absolute measurements of attenuation coefficients can be difficult. In the first lecture we saw that at boundaries between two materials of different acoustic impedances some part of the acoustic energy is transmitted and some reflected. In the case of pulse-echo measurements there will be a reflection at the transducer/material boundary of the transmitted pulse and also of the echo pulse. In addition, there will be transmission losses at the back face of the specimen. Such losses can be reduced by ensuring there is good coupling between the transducer and the specimen, and that there is a large impedance mismatch at the specimen back face. It is obvious that relative measurements of attenuation are easier to make, i.e. changes during a given test, or purely qualitative tests. In such cases the shape of the specimen, the probe and coupling are kept constant and the amplitudes of the back wall echoes can be compared. It may be possible to measure the effects of temperature changes on attenuation in this way, for example.

If one is working with different specimens of identical shape then one must ensure that they have the same acoustic impedances and same surface quality, because both these properties affect coupling and reflection losses. For example, the fact that a particular block of aluminium gives a larger back wall echo than a steel block of identical shape does not necessarily mean that aluminium possesses a lower attenuation coefficient. The cause of such an effect is often simply that there is better sound transfer in the coupling medium as a result of better acoustic impedance matching.

If one wishes to attempt absolute measurements of attenuation it is necessary to use the decrease in echo amplitude of a sequence of multiple back wall echoes. For plane wave propagation the decrease in amplitude should follow an exponential decay law and the attenuation coefficient can be calculated from the ratio of two adjacent peaks. However, it is necessary to remember that the signal will be attenuated by losses at the interface between the transducer and the specimen, and by divergence of the acoustic beam. The coupling loss can be estimated for standard transducers by using a liquid path between the transducer and the specimen surface. If the properties of the liquid are known (acoustic impedance, attenuation and path length from transducer to specimen), then the reflection losses at the boundary can be calculated and used to correct the results. For losses due to beam divergence one can either ensure that one is working in the far field of the transducer where the wave can be considered to be approximately plane, or one can make what are known as "diffraction corrections" using one of the several techniques which are described in the scientific literature.

"Diffraction correction" methods can be found in:

Bacon R.A. and Chivers R.C. (1981) Radiation coupling of a disc to a disc: a numerical approach, Acoustics Letters, 5, pp 22-29

Khimunin A.S. (1972) Numerical calculation of the diffraction corrections for the precise measurement of ultrasound absorption, Acustica, 27, pp 173-181

Khimunin A.S. (1975) Numerical calculation of the diffraction corrections for the precise measurement of ultrasound phase velocity, Acustica, 32, pp 192-200

Seki H., Granato A,. and Truell R. (1956) Diffraction effects in the ultrasonic field of a piston source and their importance in the accurate measurement of attenuation, J. Acoust. Soc. Am. 28, pp 230- 238

4.3.2 Through-transmission methods

The pulse-echo measurements of sound speed and attenuation which we have just considered are really only appropriate for solids. However, through-transmission techniques can be applied to both liquids and solids provided the material can be held in a suitable container.

An example of such a system is the materials characterisation facility which exists at the UK's National Physical Laboratory. It uses the well-known principle of through-transmission substitution. A sample of test material is prepared in the form of a parallel-sided slab, or in the case of a liquid is held in a parallel-sided container with thin end walls (typically made of mylar), and placed in a water tank between a pair of co-axially aligned transducers (or transducer and hydrophone), one of which is the transmitter and the other the receiver. Changes in signal amplitude and arrival time produced by insertion of the sample within the water path are used to derive the transmission loss (from which attenuation can be calculated) and the sound propagation speed of the material. The NPL facility allows precise and accurate manipulation of the transmitting and receiving transducers and of the test specimen in order to allow careful alignment of all components. The temperature of the water in the tank can be measured with a precision of $\pm 0.01^o C$ and the specimen is allowed to equilibrate to the temperature of the water before measurements are carried out.

The transmit-receive system consists of a broadband transducer manufactured by the Medicoteknisk Institute (Denmark) operating as a thick crystal device and driven by a Metrotek MP217 pulser, which provides the pulse used to interrogate the specimen. The pulse is detected by GEC-Marconi pvdf hydrophones of 1 mm, 4 mm or 75 mm in diameter. The peak acoustic pressures generated during measurements with the system are below 50 kPa and the bandwidth of the transmit- receive system is greater than 18 MHz.

The amplified hydrophone voltage is digitized using a Tektronix 7854 oscilloscope, waveform acquisition being followed by data transfer to a microcomputer for Fast Fourier Transform analysis and storage. All calculations are carried out in the frequency domain so that transmission losses can be obtained as a function of frequency. A counter-timer is used to provide accurate measurements of the propagation time delay which, in conjunction with the unwrapped phase data derived from the Fast Fourier Transform, is employed to calculate the frequency-dependent phase velocity of the specimen. In both cases the results are obtained by comparing the received signal for the water path alone with that obtained when the specimen is in place.

It should be noted that this method shares some of the limitations of the pulse-echo method. In particular there are reflection losses at the front and back face of the material. However, corrections can be made for these losses if the acoustic impedance of the material is known. But note that this assumes we already know the speed of sound in the specimen. In addition we will get refraction of the beam if the specimen's faces are not parallel and they are not aligned so that the faces are normal to the acoustic beam axis.

Measurement uncertainties tend to increase with frequency. We saw in the first lecture than absorption of ultrasound by materials increases with frequency. If measurements are made over a broad frequency band the received amplitudes of the higher frequency components may be very small and the quality of the measurements may be degraded by noise effects. High signal-to-noise ratios are required for good quality results.

4.4 Measurement of the scattering cross-section

Inhomogeneous materials not only absorb the acoustic wave, they also scatter it, often away from the receiving transducer. Thus it is likely that an observed attenuation of an acoustic signal in such a medium may arise from both processes. The ways in which materials scatter ultrasound can be extremely complex and it is not possible for us to study them in detail. The aim in this section of the lectures is simply to give you some idea of the difficulties involved and to make you aware of some the issues you may have to consider when you are required to characterise inhomogeneous materials.

We have already seen that sound may be reflected at boundaries when there is a mismatch of acoustic impedance, but we have tended to consider that those boundaries extend over distances equivalent to many wavelengths of the incident acoustic wave. When we use the term "scattering" rather than reflections we are usually talking about variations in the properties of the medium which are on the scale of or much smaller than the acoustic wavelength.

These scattering effects are extremely important in ultrasonic imaging, and diagnostic medical imaging in particular. If there were no scattering effects within the soft tissue organs we were trying to image there would be nothing for us to detect within the organ and we would image only the outline of the organ, where there was a major change in tissue type. The fact that we can see structure within organs indicates that scattering processes are occurring which cause the amount of energy reflected back to the transducer to vary from point to point.

4.4.1 Scattering by spheres and cylinders

For simple objects such as spheres and cylinders it is possible to predict the manner in which they scatter sound. It is found that the pattern is a function of the medium in which they are located, the speed of compressional waves and shear waves within the material, the diameter of the sphere or cylinder and the wavelength of the incident acoustic wave. The scattered field is also a function of angle.

Calculations for such simple shapes are often useful because it may be possible to consider a particular inhomogeneous material to be a large collection of spherical or cylindrical scatterers. For example, a composite material made of reinforcing fibres in a matrix may be regarded as a collection of cylinders. Similarly, heart muscle, which consists of long fibres, is often modelled as bundles of cylinders. Clearly, in such cases the scattering pattern will depend also on the angle at which the sound meets the cylinder.

Another common approach is to regard an inhomogeneous material as a collection of small scatterers which are randomly distributed and it is possible to make some theoretical calculations of expected scattering patterns for such materials. It is often assumed in such cases that the scattering arises from small fluctuations in the density and compressibility of the material, and it is this model which appears to be frequently applied to soft biological tissues.

As we have seen, scattering patterns vary with angle, and it would appear that it should be possible to characterise inhomogeneous materials by their angular scattering patterns. Several research groups both here and in the United States have attempted such measurements on biological tissues.

The material property which we try to measure in scattering experiments is the scattering cross-section. The total cross-section of a medium is the power lost to interactions of sound with the medium divided by the incident intensity. The total cross-section is the sum of the absorption cross-section and the scattering cross-section. In practice, we use finite receiving devices so that we can only detect a small fraction of the scattered power. In this case we define a differential scattering cross-section, which is the scattered power per unit incident intensity per unit solid angle per unit volume.

Scattering cross-sections are difficult to measure accurately and there are wide ranges of values quoted for particular materials in the scientific literature. Angular measurements also present special difficulties so that many experimenters try simply to measure the back-scattered power, using the kind of experimental arrangement we have seen is employed for pulse-echo measurements of attenuation and velocity. In the case of back-scattering measurements it is also relatively simple to calibrate one's measurement system, as it is possible to compare the signal obtained from the material with that obtained from a perfect reflector. Such methods of calibration are not easy to employ at other angles.

Chapter 5

Non-destructive testing using ultrasound

Quality control is essential in modern industry. Manufacturers need to be able to assess such properties as the strength, hardness and toughness of the materials they use, and an understanding of how material structure affects service performance is also necessary. Much information of this kind is provided by destructive tests on materials and manufactured components. However, there are clear advantages for manufacturers and customers if products can be assessed non-destructively.

The main use of non-destructive testing is to identify flaws and defects in components and to assist the manufacturer in deciding whether flaws are significant or not. NDT is less successful at providing information about the microstructure of materials. A range of techniques is available to the manufacturer - each appropriate to particular materials and particular kinds of defect. These techniques include magnetic particle inspection, liquid penetrant inspection, eddy current testing, x-rays, acoustic emission and ultrasonic testing.

Ultrasonic techniques can be used to detect both large and small flaws, either within a component or close to a surface. The technique is not limited to metals, but may be used with polymers and ceramics, and it lends itself both to manual and to fully automated operation. It can also be used for in-service testing of components to monitor and evaluate the growth of cracks.

The economics of the manufacturing process play an inevitable part in the designing of non-destructive testing systems. Manufacturers are not keen to reject a component if it has flaws which are too small to affect its operation. Thus there is a trade-off between the smallest flaw which an inspection system can detect and the largest flaw which the manufacturer is happy to ignore.

The American Society for Metals, in an account of ultrasonic inspection methods, define a flaw as:

> ... a detectable lack of continuity or an imperfection in a physical or dimensional attribute of a part. The fact that a part contains one or more flaws does not necessarily imply that the part is non-conforming to specification nor unfit for use.[1]

5.1 Ultrasonic NDT technology

We have already considered most of the physical concepts we shall need to understand NDT technology. We have examined the ways in which transducers are manufactured and studied the radiation patterns of simple transducer shapes.

In NDT two types of ultrasound transducer are commonly employed - normal beam probes and angled beam probes. The normal beam probe is used to transmit compressional waves into the test piece at right angles to the surface of the material. In angled beam probes, the transducer crystal is set at an angle to the surface of the test piece so that shear waves or Rayleigh waves can be transmitted into the material. In both cases the transducer is usually housed in a perspex block and backed with a damping medium. The face of the perspex block can be shaped to fit the contours of particular components.

[1] ASM Committee on Ultrasonic Inspection, Ultrasonic inspection; in Metals Handbook, Vol 11, Non-Destructive Inspection and Quality Control, 1967, pp 161-198

It is not possible to detect echoes in what is called the "dead zone". Once the transducer's element has been set vibrating by the voltage pulse it will continue to vibrate for a finite time after the pulse has ceased. It is not possible to detect a flaw during this "dead" period. Remember also that the beam spreads out beyond the point of near to far field transition. It is also attenuated by propagation through the specimen. This means that detection sensitivity is reduced far from the transducer.

We saw in earlier lectures that it is necessary to have some means of coupling the ultrasound from the transducer into the material. It is necessary to eliminate air between the transducer face and the specimen because the acoustic impedance mismatch is so great for air that most of the sound will be reflected rather than transmitted. Air can be eliminated by using a coupling medium such as water, oil, grease or pastes, such as wallpaper paste. Some soft rubber materials can also be used, particularly where coupling can be achieved by applying hand pressure to the transducer unit.

Choice of couplant depends on factors such as the smoothness of the surface of the test piece, the corrosion resistance of the material and how easy it is to clean the component after test. If the size and shape of the component are suitable, uniform coupling can be achieved by immersing the test piece completely in liquid. Uniform contact is essential between the transducer, couplant and test piece if consistent results are to be obtained. The need for couplants is one of the limitations of ultrasonic testing, as this tends to prevent its use on very hot surfaces. We saw, too, that temperatures higher than the Curie point can de-pole piezoelectric materials. Occasionally it is possible to use a cylindrical length of buffer material between the transducer and the material surface if one has to work at high temperatures.

5.2 Pulse-echo inspection

In this method of inspection short bursts of ultrasonic wave energy are transmitted into the test piece at regular time intervals. If the pulse encounters a reflecting surface such as the back face of the test piece or a discontinuity inside the test piece, a proportion of the wave energy if reflected. The amount of energy reflected depends on the size of the reflecting surface and the angle between the reflecting surface and the beam. The amount of energy reflected in a particular direction and the time delay between the initial pulse and the receipt of the echo can provide information abut discontinuities in the test piece.

Pulse-echo systems normally contain the following components:

- electronic clock;
- electronic signal generator or pulser;
- sending transducer;
- receiving transducer (may also be the sending transducer);
- echo-signal amplifier;
- display device - oscilloscope screen or X-Y plotter.

There may also be a marker circuit which can produce a secondary trace on the oscilloscope screen, usually in the form of a square wave, which is used for precise depth measurements.

Often a single transducer acts as sender and receiver, although in the so-called "pitch-catch" method of inspection, one transducer acts as transmitter and another as receiver.

The figures associated with this chapter contain a schematic diagram which shows the relationship between the main components, with the results being displayed on an oscilloscope screen.

The system operates in the following way. At regular intervals the clock triggers the signal generator, which applies a short burst of high-frequency alternating voltage (or often simply a negative-going spike), usually between 300 and 1000 volts, to the transducer. At the same time the clock activates a time-measuring circuit connected to the display device. The operator of the test equipment preselects a constant time interval between pulses. Typical repetition times are between 60 to 2000 times a second. The transducer converts the alternating voltage pulse into a pulse of mechanical vibration. The vibration is conveyed via the couplant to the test piece and travels through the material as a wave motion. When the pulse encounters a reflecting surface some energy may be reflected back to the transducer. The received signal is amplified and sent to the display device.

In theory, the maximum inspection depth is a function of the pulse repetition rate. Suppose a 10 MHz pulse of pulse length 10 cycles is transmitted at a repetition rate of 500 pulses a second. A longitudinal wave could travel approximately 12 metres in steel before the next pulse is due. The pulse could travel to a depth of six metres and return before the next pulse is transmitted. However, the depth achievable in practice may be less than the theoretical six metres owing to attenuation. Also, for the best results it is important to allow all the ultrasonic vibrations from one pulse to die out before the next pulse is transmitted. This is done by setting a pulse repetition time which would allow one pulse to traverse the test piece between five to ten times in both directions.

The display devices most commonly used in ultrasonic non-destructive testing are oscilloscopes and X-Y plotters. Three types of display are normally employed, and these are known as A, B and C scans.

5.2.1 A-scan

The most commonly used system is the A-scan display. A blip appears on the CRT screen on the left-hand side, corresponding to the initial pulse, and further blips appears on the time base, corresponding to any signal echoes received. The height of the echo is generally proportional to the size of the reflecting surface but is affected by the distance travelled by the signal and attenuation effects within the material. The linear position of the echo is proportional to the distance of the reflecting surface from the probe, assuming a linear time base. This is the normal display for hand-held probe inspection techniques. In this technique the display shows only the result achieved when the probe is located at a particular point on the surface of the test piece.

Flaw size can be determined by comparing the height of the discontinuity peak with signals from a discontinuity of known size and shape. Modern digital techniques allow the data on the screen to be recorded for later analysis, if necessary.

5.2.2 B-scan

In this technique the display shows a plot of time versus distance as the transducer is moved in a straight line across the test piece. One axis of the display corresponds to elapsed time while the other shows the position of the transducer along a line on the test piece surface. Unlike the A-scan this technique does not provide direct measurements of echo intensity. Its main use is to provide an image of an imaginary cross-section through a material, indicating the depth, position and orientation of reflecting surfaces. The main value of this technique is its ability to indicate the distribution of flaws through a cross-section of a test piece.

B-scans are, in fact, the most common way in which medical diagnostic ultrasound images are displayed. For medical applications the scanning effect may be obtained by wobbling or rocking the transducer backwards and forwards or by using a multi-element transducer and pulsing different sets of elements to produce scan lines in different positions.

5.2.3 C-scan

In this case the display on an X-Y plotter records echoes from the test piece as a function of their position within a particular area on the test piece as the probe is moved across that area. Flaws are shown superimposed on a plan view of the test piece. The technique records flaw size and position but cannot give an indication of the depth of the flaw within the test piece.

5.2.4 Calibration of probes and display equipment

Exact determination of the size, shape and orientation of flaws can be difficult owing to the large number of factors which affect the manner in which an ultrasonic wave is transmitted and reflected. Thus ultrasonic equipment is calibrated using reference blocks which have known flaws. By using test blocks it is possible to set the pulse-length and sensitivity controls appropriately. The operator uses the reference standards to adjust the controls to ensure that as many harmful flaws as possible are detected, and to indicate which signals come from insignificant flaws, so that work is not scrapped unnecessarily.

Reference blocks may be chosen which contain natural flaws similar to those likely to be in the part being inspected. The limitation here is that it is difficult to obtain different blocks which give identical responses. To overcome this, artificial flaws are introduced into test blocks by drilling holes or cutting notches. The test block may be used to set the instrument controls, when it is known as a calibration block, or it may be used to compare with the signals obtained from a component being tested, when it is known as a reference block.

5.3 Advantages and disadvantages of ultrasonic non-destructive testing

There are a number of practical constraints on the use of ultrasonics in NDT. The design of a component may be such that many parts of it are difficult to inspect owing to limited access or complex shapes. The materials themselves may be anisotropic, inhomogeneous, or have high attenuation. For reliable results inspection staff must have adequate training, and experience of the components being tested. Furthermore, ultrasonic testing requires the integration of a wide range of scientific and engineering disciplines.

Ultrasonic testing is not an automatic guarantee of the quality of a tested part. The technique requires careful, skilled use. However, compared with many other NDT techniques ultrasound has a number of advantages. They tend to be:

- superior penetrating power, allowing the detection of flaws deep inside a component;
- high sensitivity, permitting the detection of extremely small flaws;
- greater accuracy than other NDT methods in determining the position, size, orientation and shape of internal flaws;
- reliability (no moving parts);
- only one surface of a component need be accessible;
- electronic operation provides almost instantaneous indication of flaws (typically less than a millisecond); testing can be automated and used for production monitoring and process control;
- not hazardous to operators and other personnel; does not affect nearby equipment and materials;
- portability;
- Low cost, especially for multiplexed or mass-produced sensors.

Disadvantages are:

- manual operation requires careful attention by experienced technicians (part of your coursework involves studying this);
- wxtensive technical knowledge is required for the development of test procedures;
- components which are rough, irregular in shape, very small or thin, or not homogeneous are difficult to inspect;

- discontinuities immediately beneath the surface of a component may not be detectable (dead time)
- couplants are needed to provide effective transmission of wave energy from the transducer to the test piece;
- does not work above the Curie temperature of the transducer;
- reference standards are necessary, both for calibrating equipment and characterising flaws.

Exercise

In the light of what has been set out above, read through the Krautkramer document `$phlib/2us/krautkramer.pdf` which sets out an introduction to the basic inspection techniques of ultrasonic NDT and make your own additional notes.

5.4 Flow measurements by Doppler ultrasound

A key method of non-destructive testing that will not be considered further here is measurement of flow velocity using the Doppler effect. This is the subject of one of the projects and will be covered in more detail in the Level 2 Medical Acoustics module.

Chapter 6

Measuring ultrasonic power and intensity

If we are interested in assessing the energy generated by an ultrasound beam, it is possible to use thermal measurement techniques, which are especially useful in homogeneous materials, where scattering is negligible and the sound energy is absorbed by the medium. The transfer of energy results in a temperature rise which can be measured using two different approaches.

6.1 Calorimetry

This method is an apparently simple way of measuring the acoustic power output produced by ultrasound transducers. In theory, it is necessary only to ensure that all of the acoustic energy is converted to thermal energy by absorption in an absorbing liquid such as castor oil or carbon tetrachloride (solids are also used). The power may then be calculated from the resulting increase in temperature. In practice, thermal losses are difficult to control and the temperature rise may be difficult to measure accurately, particularly from transducers with lower power output, such as those used in medical diagnosis where power may be as low as a few milliwatts.

There are two main types of calorimeter.

- The liquid in the absorbing reservoir is stirred during irradiation with ultrasound and the final mean temperature rise is determined. Problems arise because of the difficulty of stirring viscous liquid and the slow response time of the device.
- The most commonly used arrangement is the continuous flow calorimeter in which liquid is passed through an absorption chamber at a fixed rate and the input and output temperatures are monitored.

The devices are usually calibrated using an electrical heater supplying a precisely known amount of electrical energy to produce the same average temperature rise. This technique measures only total power and hence gives no spatial or temporal resolution.

6.2 Thermocouples

Local measurements of acoustic intensity may be made using small thermocouples embedded in an absorbing medium. An early device (Fry W.J. and Fry W.B. 1954, Determination of absolute sound levels and acoustic absorption coefficients by thermocouple probes, J. Acoust. Soc.Am. 26, pp 294-317) used a thermocouple in castor oil between two polyethylene diaphragms. Once the ultrasound is turned on there is a large initial rise in temperature which is usually attributed to a viscous interaction between the medium and the thermocouple wire (the motion of the wire is out of phase with the motion of the liquid). This is followed by a slow, almost linear rise in temperature owing to the absorption of ultrasound by the medium surrounding the wire. The gradient of the second region gives a measure of the local time-averaged intensity.

The attraction of the technique is its simplicity coupled with the fact that thermocouples can be made as small as ten microns in diameter, allowing good spatial resolution to be obtained. However, the slow response time means that the time variation of pulses cannot easily be resolved. In addition, fine thermocouples are difficult to manufacture and their performance may not be reproducible.

In principle, for a liquid whose absorption coefficient is known, the device could produce absolute measurements of intensity, but in practice they are calibrated using a small sphere radiometer (see next section of lecture notes).

One technique which is being developed by various experimenters, including the National Physical Laboratory, is to embed thermocouples (both fine wire and thin film designs) in tissue-mimicking gels, in order to map the spatial distribution of temperature rises produced by diagnostic ultrasound scanners.

6.3 Radiation force measurements

The propagation of ultrasound through a medium is associated with a simultaneous transfer of momentum. Momentum is transferred to any absorbing or reflecting object which intercepts the ultrasonic beam, and this is revealed as a radiation force acting on the obstacle. The measurement of this force provides the basis for a number of acoustic intensity and power measuring systems. The force acually depends on the time-averaged intensity of the ultrasound beam. There are two basic types of measurement system depending on how much of the beam is intercepted by the obstacle or target.

- Large target: the target intercepts the whole of the ultrasonic beam to provide a value of the total acoustic power.
- Small target: the target is much smaller than the spatial extent of the beam and yields the time-averaged values of the local intensity.

6.3.1 Large target: the radiation force balance

The principle behind the force balance is the measurement of the force experienced by a target which totally intercepts the ultrasonic beam. The force, F, is related to the acoustic power, W, by the expression:

$$F = \frac{hW}{c} \tag{6.1}$$

where c is the speed of sound in the medium through which the sound propagates, and h depends on the geometry of the target.

An idea of the sensitivity needed in such devices is provided by the fact that for plane wave propagation a total power of one milliwatt will produce a force of 0.69 micronewtons on an absorbing target.

Many different types of radiation force balance have been described in the scientific literature and you will see at least two of them during your visit to the National Physical Laboratory.

6.3.2 Small target: the suspended sphere radiometer

The force experienced by a small target can be used to determine local values of acoustic intensity. The target should ideally be small in comparison with the spatial variations of pressure in the field, and is normally a few wavelengths in diameter.

A small sphere is suspended from a thin filament in a manner resembling a pendulum. The acoustic beam produces a horizontal deflection in the position of the sphere.

The radiation force is given by:

$$F_r = \frac{mgd}{(L^2 - d^2)^{1/2}} \tag{6.2}$$

where m is the mass of the sphere, g is the acceleration due to gravity, d is the horizontal deflection of the sphere, and L is the length of the filament.

If simple spherical targets are used it is relatively easy to relate the force to the local value of the acoustic intensity. We can derive a quantity , Y_p, the acoustic radiation force function, given by:

$$Y_p = \frac{F_r}{\pi a^2 \overline{E}} \tag{6.3}$$

where a is the sphere radius and, $\overline{E}$ is the mean energy density of the field, so that Y_p is the radiation force acting on the sphere per unit cross-sectional area per unit mean energy density.

The time-averaged intensity of a plane progressive wave is given by $I = \overline{E}c$, so that the measured intensity is:

$$I = \frac{F_r c}{\pi a^2 Y_p} \tag{6.4}$$

The reliability of the calculations depends on how well the experimental apparatus mimics theory. Filaments must be thin enough to be non-perturbing and the way in which they are attached should not alter the spherical nature of the target. Y_p is also a function of the wavelength and there is a complex relationship between it and ka.

The technique is relatively insensitive so that it is not appropriate for measurements on diagnostic medical equipment, for example. Very small deflections are difficult to measure, as are the effects of pulses, rather than continuous waves.

Chapter 7

Non-linear effects in ultrasonic measurements

So far we have considered only linear propagation of acoustic waves, and have often assumed that we are working with continuous waves of a single frequency. Where we have been concerned with pulses, we have assumed that the pulse can be decomposed into a spectrum of sine waves propagating with different amplitudes, frequencies and phases.

In reality, this is an approximation which only applies for acoustic disturbances of very small amplitude. In many practical situations we generate signals of substantial amplitude (many tens of atmospheres in medical diagnostic ultrasound, for example). In such a case our linear approximations no longer apply.

We saw in the first lecture that local changes in density cause local changes in pressure and that the acoustic pressure, p, is related to the condensation, s, where $s = (\rho - \rho_o)/\rho_o$. The equation of state which describes this relationship is:

$$p = P - P_o = As + \frac{Bs^2}{2} + \cdots \tag{7.1}$$

where P is the absolute pressure, P_o is the ambient pressure and A and B are constants. If we use only the first term to derive the acoustic wave equation, we obtain a linear wave equation. If we include the second term, we obtain a non-linear equation.

Suppose that we launch a high amplitude sine wave from a transducer into a relatively loss-less medium, such as water. We find that as the wave propagates it becomes more and more distorted. This is because the compressions travel faster than the rarefactions, so that the wave comes more and more to resemble a saw tooth shape rather than a sine wave. We can consider this process as a transfer of energy from the original sine wave to its higher harmonics. However, we also saw that higher frequencies are attenuated more rapidly than lower frequencies. After propagating some distance in the medium, the higher harmonics become progressively more attenuated, so that the waveform eventually becomes less distorted.

It is physically impossible to obtain an exact saw tooth shape as this would imply that an infinite range of pressures existed at a single point or at one moment in time. Nevertheless, it is not difficult to generate highly shocked wavefronts which closely resemble the saw tooth shape.

When one is working with linear theoretical models, such as when one is measuring the attenuation, sound speed and scattering cross-section of a material, it is essential to work at pressures which are low enough to avoid non-linear phenomena.

The frequency spectrum of a saw tooth contains harmonics whose amplitudes are proportional to the reciprocal of the harmonic in question, that is, if we regard the fundamental frequency as having an amplitude of 1, then the second harmonic has an amplitude of 1/2, the third has an amplitude of 1/3 and so on. If we suspect the presence of non-linear distortion, an examination of the frequency spectrum of the signal may help us to identify the phenomenon, especially if there are clearly defined harmonics of the fundamental with amplitude ratios of approximately the magnitudes described above.