Level 3: Medical Imaging
I Magnetic Resonance Imaging

1. Recommended material for further study

- Probably the best introductory resources from a Physics point of view are now web-based. I suggest that you browse first on the “links” page of the International Society for Magnetic Resonance in Medicine (ISMRM), located at
  
  http://www.ismrm.org/mr_sites.htm

  Of the various sites listed here, I recommend:

  “The Basics of MRI”, by Joseph Hornak, located at
  
  http://www.cis.rit.edu/htbooks/mri/

- There appears to be a shortage of good introductory textbooks to NMR and MRI from a physics rather than clinical point of view. You may have to do a little research by yourselves to find a book in the library that you “get on with”. Two possible books to try are:


  However, neither of these really have a very good Physics content.

- Good textbooks at a higher level are:


- An introductory textbook coming from the chemistry point of view (spectra, chemical analysis, etc.) is:

2. Introduction

2.1 What is Magnetic Resonance Imaging (MRI)

- MRI is a method of *medical imaging*, i.e., taking pictures of the insides of people to diagnose illnesses.
- Because it does not use ionising radiation, a properly conducted MRI scan is (as far as we can establish) completely safe.
- In contrast to simple X-ray imaging, we can select *slices* through the patient with any desired orientation and we can obtain images with a *wide range of contrast*.
- MRI is used to image many different parts of the body, not just the brain. In fact, we can image any bit of the body that will fit into the scanner.
- We can obtain images at a variety of different *resolutions* to look at different sized objects.

2.2 How Does MRI Work?

- The sample (i.e., the patient) is placed in a very strong magnetic field.
- The human body contains a very large amount of hydrogen, in the form of water molecules.
- Because of a quantum-mechanical property called *spin*, each hydrogen nucleus behaves like a tiny bar magnet. It can line up in two ways, *with* and *against* the field $B$. The two orientations have different *energies*.

![Figure 2.1: Head images obtained using MRI. Taking pictures of the brain is the number one use for MRI in hospitals. We can take slices in any desired orientation; (a) is called a “transverse” (or “axial”) slice and (b) is a “sagittal” slice.](image)
2.2 HOW DOES MRI WORK?

- By transmitting a pulse of radio waves at a frequency corresponding to the difference in energy between the two energy states, we can excite nuclei from the lower to the higher level.

- The frequency of the absorbed radiation depends on the magnetic field across the sample by the fundamental equation of MRI:

  \[ f = \gamma B \]

  \( f \) is the angular frequency of the NMR signal. \( \gamma \) is a constant (42 MHz / T). \( B \) is the magnetic field.

- An NMR spectrometer, which, at its heart, is simply a glorified radio receiver, detects and discriminates between radio signals at different frequencies. NMR spectrometers are used widely in chemistry to analyse unknown compounds.
because the frequency depends on chemical structure.

- The sample (in medicine, the patient!) is placed in a very strong magnetic field. Most clinical imagers have field strengths between 0.5 T and 3 T.

- The magnet is normally a superconducting solenoid.

Fig. 2.4: Arrival of the new 3 T magnet at the VA Hospital in Gainesville FL

- The reason it is so bulky is that the superconducting wires must be surrounded by liquid helium to keep them cold and then, the whole lot must be placed in a giant “vacuum flask” to stop head getting in.

- If we deliberately make the magnetic field change with distance, then the frequency of the signal depends on position. The vital extra ingredient that an MR imager has over a chemistry spectrometer is the ability to make the magnetic field change with position. This is achieved using a set of magnetic field gradient coils. There is one for each of the directions, \( x \), \( y \) and \( z \).

\[ B = B_0 + x G_x \]

Object in magnet

1-D “image” or object “profile”

Figure 2.5: Principle behind 1-D imaging. The 2-D version is similar.
3. Nuclear Magnetic Resonance (NMR)

3.1 Introduction

- NMR stands for Nuclear Magnetic Resonance.
- NMR experiments allow us to find out physical, chemical and spatial information about a sample.
- NMR uses the transitions between two spin states of a nucleus (typically the hydrogen nucleus), which correspond to different directions of the nuclear magnetic moment.
- NMR is currently one of the most widely used tools in the physical sciences. Its major applications are:
  
  (i) **NMR Imaging** (also called MRI) — “the” method for most modern brain scans and very widely used in hospitals;
  (ii) **NMR Spectroscopy** — an invaluable tool in analytical chemistry;
  (iii) **Materials Characterisation** — from oil-bearing rocks to paint to breakfast cereals, NMR techniques allow us to find out a variety of physical properties

3.2 The Microscopic Approach to Understanding NMR

   This is the semi-classical energy level approach, suggested by Rabi’s experiment and followed by Purcell *et. al.*

3.2.1 Spin Quantisation

- Many nuclei possess a property called spin.
- The spin may be regarded as an angular momentum, analogous to that possessed by a spinning top.
- The spin is quantised, with quantum number $I$, and the spin angular momentum is

\[ I(I + 1). \]  

[3.1]

- There is a useful empirical rule for finding $I$ for a given nucleus with $p$ protons and $n$ neutrons:
  
  (i) If $p$ and $n$ are both even, then $I = 0$.
  (ii) If $(p+n)$ is odd, then $I$ is a half-odd-integer ($1/2, 3/2, 5/2$, etc.).
  (iii) If $(p+n)$ is even, the $I$ is an integer.
For further details, see textbooks on specific nuclear shell models.

- In NMR (and particularly imaging), we are interested in mainly spin-1/2 nuclei. Typical examples include $^1\text{H}$, $^{13}\text{C}$ and $^{31}\text{P}$.

- Applying a magnetic field *defines* a measurement directions. Look up the Stern-Gerlach experiment in a quantum mechanics textbook to find more details about this. Normally, we take the direction defined by the magnetic field as the $z$-axis. This means the nuclear spins are quantised along $z$.

\[ m_z \in \{-\frac{1}{2}, \frac{1}{2}\} \quad \text{[3.2]} \]

For a spin-1/2 particle, such as the hydrogen nucleus (proton), $m_z$ has just two values, $-1/2$ and $+1/2$.

### 3.2.2 Magnetic Moment

- Because of its spin, a nucleus has an associated magnetic (dipole) moment. Classically, a point charge orbiting about a given axis has

\[ I \mu = \frac{e q}{2} \quad \text{[3.3]} \]

- When we look instead at *spin* rather than orbital angular momentum, we must include another factor. A more complicated model is needed to calculate this, which we shall not discuss.

\[ I \mu = m q g \quad \text{[3.4]} \]

where $g$ is the so-called *nuclear g-factor*. Often, we lump all the constants together and write simply

\[ I \mu = \gamma \quad \text{[3.5]} \]

where the new constant $\gamma$ is called the *gyromagnetic ratio* (sometimes “magnetogyric” ratio). This equation also reflects the fact that spin, and hence magnetic moment, is a vector quantity. $\gamma$ is found experimentally and is different for each nucleus. Table 3.1 shows the properties of a number of nuclei.

### 3.2.3 Energy Levels

- Classically, the energy of a magnetic dipole $\mu$ in a magnetic field $B$ is given by

\[ E = \mu B \quad \text{[3.6]} \]

referred to the zero of an otherwise identical particle in the same place with $\mu = 0$. 

6
3.2 The Microscopic Approach to Understanding NMR

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^1\text{H}$</th>
<th>$^2\text{H}$</th>
<th>$^7\text{Li}$</th>
<th>$^{13}\text{C}$</th>
<th>$^{14}\text{N}$</th>
<th>$^{15}\text{N}$</th>
<th>$^{17}\text{O}$</th>
<th>$^{19}\text{F}$</th>
<th>$^{23}\text{Na}$</th>
<th>$^{31}\text{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin</td>
<td>1/2</td>
<td>1</td>
<td>3/2</td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>5/2</td>
<td>1/2</td>
<td>3/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$\gamma$ / rad s⁻¹T⁻¹</td>
<td>2.675</td>
<td>0.410</td>
<td>1.037</td>
<td>0.673</td>
<td>0.193</td>
<td>0.272</td>
<td>0.365</td>
<td>2.517</td>
<td>0.708</td>
<td>1.081</td>
</tr>
<tr>
<td>Natural Abundance</td>
<td>99.985</td>
<td>0.015</td>
<td>92.58</td>
<td>1.108</td>
<td>99.63</td>
<td>0.365</td>
<td>0.037</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Sensitivity (relative to $^1\text{H}$)</td>
<td>$=\gamma A(I=\pm 1)$</td>
<td>1</td>
<td>$9.65 \times 10^{-3}$</td>
<td>$0.293 \times 10^{-2}$</td>
<td>$1.59 \times 10^{-3}$</td>
<td>$1.01 \times 10^{-3}$</td>
<td>$1.04 \times 10^{-3}$</td>
<td>$2.91 \times 10^{-2}$</td>
<td>0.833</td>
<td>$9.27 \times 6.59 \times 10^{-2}$</td>
</tr>
<tr>
<td>Sensitivity × Natural Abundance</td>
<td>1</td>
<td>$1.45 \times 10^{-6}$</td>
<td>$0.271 \times 10^{-4}$</td>
<td>$1.76 \times 10^{-4}$</td>
<td>$1.01 \times 10^{-4}$</td>
<td>$3.85 \times 10^{-6}$</td>
<td>$1.08 \times 10^{-5}$</td>
<td>0.833</td>
<td>$9.27 \times 6.59 \times 10^{-2}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: NMR properties of a number of stable nuclei

- This means that if there are two spin states (i.e., $m_z = \pm 1/2$), then there are two energy levels — see Fig. 3.1.

- In NMR, we normally have a strong static field along the $z$-axis, which we call $B_0$. Hence, if $E_\uparrow$ and $E_\downarrow$ are the energies of the spin-up and spin-down states (i.e., pointing with and against $B_0$), then

  \[ \Delta E = \gamma \mu_0 = \gamma B_0. \]  

  and, similarly, $E_\downarrow = +\gamma B_0 / 2$.

- The gap between the energy levels is

  \[ \Delta E = \gamma \omega_L = \gamma B_0. \]  

  where $\omega_L$ is called the Larmor (angular) frequency.

- As usual, we can have transitions between the energy levels. These are accompanied by the absorption or emission of quanta of energy of size $\hbar \omega_L$. One interpretation of the physical origin of the NMR signal is that it corresponds to the quanta emitted. However, the situation is in fact more complicated than this.

- In nuclei with $I > 1/2$, there are more than two different energy levels. This means that more than one different transition is possible. These nuclei will have a number of different resonance frequencies.

- However, for the majority of imaging applications, we look at spin-1/2 nuclei, mainly $^1\text{H}$ and $^{19}\text{F}$. 

7
The thermal equilibrium distribution of nuclei between the two energy levels is given by a Boltzmann distribution:

\[ E = \pm \gamma \hbar B_0 / 2 \]

When placed in a magnetic field, the sample will have a net magnetisation (or magnetic moment per unit volume \( M \)), because more nuclei are in the stable spin-up configuration. \( M \) can be calculated by simple algebra:

\[ M = \mu_z \left( \frac{1}{1 + e^{-\Delta E/kT}} - \frac{e^{-\Delta E/kT}}{1 + e^{-\Delta E/kT}} \right) \]

Now we know that \( n^+ + n^- = n \), the total number of nuclei per unit volume. Using this and Eq. [3.9], we get the following expression for \( n^+ \):

Using Eqs. [3.9] – [3.11], we can now find \( M \):
3.2 The Microscopic Approach to Understanding NMR

The net magnetisation is important, because it is only the excess of nuclei which contributes to the observed signal.

Under typical NMR imaging conditions, $B_0 \sim 1$ T, $T \sim 298$ K and, if we are looking at protons $γ \approx 2.7 \times 10^8$ rad s$^{-1}$ T$^{-1}$. This means $n^\uparrow - n^\downarrow \approx 3 \times 10^{-6}$. I.e., in proton NMR, we get a signal from only three nuclei in every million and the net magnetisation is very small. This means that NMR measurements give very small signals.

When making NMR measurements, the weak signals that we receive are overlaid by thermal noise. Many NMR techniques are hampered by a low signal-to-noise ratio (SNR).

When a radio-frequency magnetic field is applied to the sample in a magnetic field, it causes changes in the populations of the two levels. Spins are promoted from the low to the high energy state.

The group of Ed Purcell first observed the NMR phenomenon by finding an energy absorption which took place at the Larmor frequency

\[ n \left[ \frac{e^{+\Delta E/2kT} - e^{-\Delta E/2kT}}{e^{+\Delta E/2kT} - e^{-\Delta E/2kT}} \right] \cdot \mu_z \]

\[ = n \frac{\sinh (\Delta E / 2kT)}{\cosh (\Delta E / 2kT)} \cdot \mu_z \]

\[ = n \tanh (\gamma B_0 / 2kT) \cdot \frac{\hbar}{2} \quad [3.12] \]

3.3 The Macroscopic Approach to Understanding NMR

This is the approach taken by Felix Bloch et al. Instead of looking at energy levels of nuclei on the microscopic scale and thinking about the relative populations of different levels, we look at the magnetic moment as a vector and the vector sum of the moments of all the individual nuclei.

3.3.1 Free Precession

Let us consider the magnetic moment of the nucleus to be like a tiny bar magnet “sitting” in a magnetic field $\mathbf{B}$ — see Fig. 3.2. Classically, such a magnet would feel a torque

\[ [3.13] \]
Newton’s Second Law of Motion tells us that if there is an unbalanced torque, then there must be a change in the angular momentum of the system.

\[ T_L = \frac{d}{dt} \omega. \] \[ 3.14 \]

This is just the equivalent of \( F = ma = \frac{dp}{dt} \), where \( p = mv \), but for the case of circular motion.

- For a nucleus, the only contribution to the angular momentum comes from its spin. Hence, combining Eqs. [3.13] and [3.14] we obtain the fundamental equation of NMR:

\[ \boldsymbol{\mu} \times \mathbf{B}_0 = \frac{d}{dt} \gamma. \] \[ 3.15 \]

This is an equation describing circular motion. The vector \( \boldsymbol{\mu} \) moves with an angular velocity (angular frequency) given by

\[ \omega = -\gamma. \] \[ 3.16 \]

- This is the equation of motion for an isolated (i.e., “free”) spin in a magnetic field. The motion is called free precession. Notice that the motion is in a plane perpendicular to \( \mathbf{B} \) and that the angular frequency has the same magnitude as that described in Eq. [3.8].

- We define the magnetisation of a sample as its total magnetic moment per unit volume

\[ 3.17 \]
3.3 The Macroscopic Approach to Understanding NMR

3.3.2 The Vector Model

- Each individual nucleus has a spin $I$ and corresponding magnetic moment $\mu$ which can take only a certain restricted set of orientations. $I_z = \pm \hbar / 2$, whilst $I_x$ and $I_y$ may take any values. This means that $I$ and $\mu$ are confined to cones.

- Quantum mechanics tells us that we cannot observe all the components of $I$ or $\mu$ simultaneously. If we try to measure $I$, then the wavefunction “collapses” to give us $\pm \hbar / 2$ for the component along the direction of measurement. Remember: in NMR our large, static field $B_0$ defines the direction of measurement, which we take to be along $+z$ by convention. Thus, we can never measure the $x$- or $y$- components of magnetisation for a single spin.

- This is not the case for the macroscopic magnetisation $M$. It should be obvious that our earlier semi-classical equation for the motion of a single spin vector can be extended to

\[ B \times = \gamma M dt \] \[ [3.18] \]

and because $M$ is a “classically large” quantity, we can in principle measure its $x$, $y$ and $z$ components.

- $M_z$ is the resultant magnetisation in the $z$-direction of all the different spins — see Fig. 3.3. Eq. [3.12] tells us what this is at thermal equilibrium. We shall call this value $M_0$. It turns out that by exciting different numbers of spins to the higher energy level by different amounts of radio-frequency energy, we can obtain values of $M_z$ anywhere between $+M_z$ and $-M_z$. 

![Figure 3.3: The thermal equilibrium distribution of spins leads to a net magnetic moment](image-url)
• At thermal equilibrium, the values of \( \mu_x \) and \( \mu_y \) for different nuclei are randomly distributed. This means that \( M_x \) and \( M_y \) are initially zero. However, the effect of a radio-frequency field is to introduce a phase coherence between the precession of the different spins, leading to non-zero values of \( M_x \) and \( M_y \).

• This model, considering the combined effect of the ensemble of spins as a single vector is known as the vector model. It is the basis for almost all NMR imaging theory.

3.3.3 Forced Precession and the Rotating Frame

• At thermal equilibrium, \( M = M_0 \, k \) points along the same direction as \( B_0 \) and so

\[
[3.19]
\]

This is the meaning of equilibrium.

• We are now going to consider excitations by a radio-frequency magnetic field, which take the system away from equilibrium. We represent the RF field by a second vector \( B_1 \), which is time-varying and, specifically, rotating at the Larmor frequency \( \omega_L \).

\[
[3.20]
\]

This equation represents a vector of length \( B_1 \) rotating in the plane perpendicular to the main field \( B_0 \) at an angular velocity \( \omega_L \).

• Now imagine that you are “sitting” on this vector going round. It is equally valid to think of yourself as stationary, while the laboratory moves backwards! This is called setting up an alternative frame of reference (c.f. special relativity).
In the rotating frame \( B_1 = B_1 \hat{i'} \), where \( \hat{i'} \) is the basis vector corresponding to the \( x' \)-axis of the rotating frame. We shall state without proof that, in this frame, the motion of the net magnetisation is given by

\[
\dot{M} = B_1 \gamma \frac{d}{dt}. \tag{3.21}
\]

You should by now recognise this as an equation of precession.

- Thus, in the rotating frame, the effect of the RF pulse is simply to cause a precession of the magnetisation around the \( x' \)-axis. This is sometimes called *forced precession*.

### 3.4 Relaxation

- In any “real” spin system that we can observe in the laboratory, the spins are not isolated; they interact with each other.

- Interactions occur by means of *fluctuating magnetic fields*. These are caused by the magnetic moments of other nuclei, which are in random motion.

- To work out the effects in full needs some in depth quantum mechanics. We shall look at things on a much simpler, phenomenological level. When an RF field is applied at resonance, it has two effects:

  (i) It changes the relative populations of the spin \( \uparrow \) and spin \( \downarrow \) states.

  (ii) It creates a phase coherence between \( \mu_x \) and \( \mu_y \) for different spins, to give non-zero values of \( M_x \) and \( M_y \).

- Two important relaxation processes occur, which we call \( T_1 \) and \( T_2 \) relaxation. \( T_1 \) relaxation *restores* the equilibrium population and \( T_2 \) relaxation destroys the phase coherence created by the pulse.

#### 3.4.1 \( T_1 \) = Spin-Lattice (Longitudinal) Relaxation

- When the system has been excited by an RF field, it is no longer in thermal equilibrium. More of the higher energy levels are occupied than normal. This corresponds to the \( z \)-component of \( M \) being *less* than \( M_0 \).

- Spin-lattice relaxation involves a *transfer of energy* out of the spin system into the surroundings. These “surroundings” are often referred to as “the lattice”, hence the name. In terms of the vector model, spin-lattice relaxation corresponds to a “growing back” of the magnetisation along the \( z \)- (or *longitudinal*) axis. Hence there is an alternative name: “longitudinal relaxation”.

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13
Spin-Lattice relaxation is an exponential recovery process and the relaxation time $T_1$ tells us how quickly this growing back occurs. The differential equation controlling this behaviour is

$$M_z = M_0 (1 - e^{-t/T_1})$$

and you should verify that the solution to this equation is

$$M_z = M_0 e^{-t/T_1}$$

The figure above shows the situation where $M_0(0) = 0$ and hence

$$M_z = M_0 e^{-t/T_1}$$
3.4.2 \( T_2 = \text{Spin-Spin (Transverse) Relaxation} \)

- Spin-spin processes are interactions between spins — hence the name — in which no energy is lost to “the lattice”. The only result is a randomisation of the spin phases. Small changes in magnetic field at the site of a spin cause it to speed up or slow down, so getting out of phase with its neighbours, which see slightly different magnetic fields.

- Spin-spin relaxation involves not a change in energy of the spin system as for \( T_1 \), but a change of entropy.

- In terms of the vector model, spin-spin relaxation corresponds to a shrinking of \( x' \)- or \( y' \)-magnetisation. \( x' \)- and \( y' \)-magnetisation are often collectively called “transverse” magnetisation; hence the alternative name “transverse relaxation”. In the figure, we have shown only a \( y' \)- component of transverse magnetisation, but in general, there may be an \( x' \)-component as well.

- Spin-spin relaxation is an exponential process and the relaxation time \( T_2 \) expresses the time taken for the magnetisation to fall to \( 1/e \) of its value at \( t = 0 \).

\[
\frac{dM}{dt} = \frac{2}{\gamma} \left( \frac{M_0 - M_z}{T_1} \right) \mathbf{k} - \frac{(M_{x'} + M_{y'})}{T_2}
\]

\[ \uparrow \quad \uparrow \quad \uparrow \]

forced precession  longitudinal relaxation  transverse relaxation

This is the most important equation in the course. All MRI is based on this!

-Whilst the original equation, containing just the precession term described a vector with constant length \( |M| = M_0 \), the new equation describes a vector which can have any length between 0 and \( M_0 \).

-This equation is called the \textit{Bloch Equation} after Felix Bloch, its discoverer.
3 NUCLEAR MAGNETIC RESONANCE (NMR)

3.5 Pulsed NMR

3.5.1 The Pulsed NMR Experiment

- In 1966, Ernst and Anderson introduced the concept of a radio-frequency (RF) pulse. Richard Ernst was awarded the Chemistry Nobel Prize in 1991 for his achievements in NMR.
- An NMR pulse is simply the application of a strong $B_1$ field for a very short time $\tau$.
- $\tau$ is short enough so that we can ignore relaxation during the pulse, i.e., $\tau \ll T_1, T_2$. Typically for water, $T_1 \sim 1\,\text{s}$ and $\tau \sim 100\,\mu\text{s}$. This means that during the application of the pulse $T_1$ and $T_2$ have a negligible effect and can be ignored.
- The Bloch equation reduces to

$$\frac{d}{dt}M = -\gamma B_1 \tau \hat{z}$$

in the rotating frame. This is a rotation about the $x$-axis, with angular frequency

$$\omega = -\gamma B_1 \tau$$

The negative sign tells us, in this case, that the rotation is in the direction from $+z$ to $+y$ and a simple diagram shows that at the end of a pulse of duration $\tau$:

$$M_x' = \cos(\gamma \tau), \quad M_y' = \sin(\gamma \tau)$$

Note that:

(i)

(ii)

(iii) the convention for describing the angle may be confusing. Normally for a rotation in the $xz$-plane, we would measure positive angles as anticlockwise with zero corresponding to a direction along the $x$-axis. In this particular case, historical convention is that $\theta$ is zero in the original orientation of the $M$, i.e., along the $z$-axis.

If we ignore relaxation, then after the pulse, this magnetisation just “stays there” if we are in the rotating frame of reference.

3.5.2 Detection of the NMR Signal

- If we look in the lab frame of reference, the magnetisation is now rotating at the Larmor frequency:
3.5 PULSED NMR

\[ M = \cos(\theta) + \sin(\theta) \]

\[ V_e^{(w_c)} \]

\[ B_0 \]

\[ B_1 \]

\[ M \]

\[ \omega_L = -\gamma B_0 \]

\[ \phi_0 \]

\[ \phi_0 \] is an arbitrary constant, corresponding to the phase of our detector system. We can set it to anything we like by modifying the electronics of the detector.

- The reason we are able to detect a signal is because the rotating magnetisation induces an EMF in the detector — see Fig. 3.6.

3.5.3 Demodulation

- Signals at the Larmor frequency are difficult to work with. We cannot easily transform them into digital data.

- Using an analogue electrical circuit, we shift the angular frequency down by \( \omega_L \).

- The NMR signal, which we will call \( S \), is called the free induction decay or FID, because the signal is an EMF induced by the free precession of the magnetisation, which is decaying due to \( T_2 \) processes.
3 NUCLEAR MAGNETIC RESONANCE (NMR)

\[ n(\omega) = \frac{n_0 \pi}{\omega_{1/2}} \cdot \frac{1}{1 + (\omega / \omega_{1/2})^2} \cdot \left(1 + e^{-t/T_2}\right). \]  \[ n(\omega) = \frac{n_0 \pi}{\omega_{1/2}} \cdot \frac{1}{1 + (\omega / \omega_{1/2})^2}. \]

\[ \text{normalising factor} \]

3.6 \( T_2^* \) Relaxation

- Let \( n(\omega) \) be the number density of nuclei with the precession angular frequency different by an amount \( \omega \) from the demodulation frequency. Typically, we assume that the variation in \( n(\omega) \) is a Lorentzian function. However, this is a very phenomenological approach, justified by what we see in practice for magnets.

Each little “group” of spins (isochromat) precessing at offset \( \omega \) contributes a signal.

\[ dS(t) = \frac{M_0 \sin \theta}{n_0} \cdot n(\omega) \cdot d\omega \cdot e^{i\omega t} \cdot e^{-t/T_2}. \]  

Figure 3.7: Distribution of Larmor frequencies when an inhomogeneous magnetic field is present. Note that we are using the definition of \( \omega \) employed in Section 4, i.e., the demodulated angular frequency.
The total signal is just the integral of this expression over all $\omega$.

$$S(t) = M_0 \sin \theta \cdot e^{-t/T_2} \int_{-\infty}^{+\infty} n(\omega) e^{i \omega t} d\omega. \quad [3.32]$$

- The final part of the expression is simply the inverse Fourier transform of $n(\omega)$. It turns out — see mathematical tables — that the inverse FT of a Lorentzian function is an exponential decay, whose time constant (which we shall call $T_2'$), whose width is related to the width of the Lorentzian curve.

$$\text{[3.33]}$$

The effect of a magnetic field inhomogeneity is to cause a signal decay. The assumption of a Lorentzian function for $n(\omega)$ gives an exponential decay.

- Because of this assumption of an exponential decay, the inhomogeneities in the magnetic field are simply equivalent to an extra contribution to the normal $T_2$ relaxation. It is often very useful to include the two terms into one:

$$\text{[3.34]}$$

where

$$\text{[3.35]}$$

- The less homogeneous the field, the shorter is $T_2^*$ and the quicker the signal decays.

### 3.7 Spin Echoes

- In 1950, Hahn showed that the two contributions to $T_2^*$ are qualitatively different:

$$\text{[3.36]}$$

- This means that the length of time over which the signal can be observed is not limited by inhomogeneities in the magnetic field.
The mechanism by which we are able to reverse the effects of $T_2'$ is called *refocusing*.

Figure 3.9 shows the signal changes that occur in what is called a spin echo experiment. The sequence of events is:

1. A 90° pulse
2. A delay $T_E/2$ — During this time we observe a decaying FID.
3. A 180° pulse
4. A further delay $T_E/2$ — During this period, we observe the signal growing back, but in the opposite direction — a virtual mirror image of the previous decay. At a time $T_E$, the signal reaches a maximum and then starts to decay again. This maximum is called a spin echo.

Why does this happen? Figure 3.8 explains.

1. The 90° pulse rotates all the magnetisation along the $+y'$-axis.
2. The $y'$-magnetisation created by the 90° pulse consists of different groups of spins (isochromats) which have different precession (Larmor) frequencies because of the inhomogeneity in $B_0$. In a frame rotating at $\omega_0$, the central Larmor frequency, some vectors rotate forwards and some backwards, whilst the isochromat with precessional frequency $\omega_0$ stays still.
3. The net transverse magnetisation is the vector sum of all these isochromats. During the first $T_E/2$ period, the vectors get out of phase. This means that the vector sum decreases in magnitude and so the signal we observe decays. This is a pictorial representation of what we mean by the integral of Eq. [3.32].
4. The 180° pulse flips all the isochromats about the $x'$-axis. Now the “fast” spins are behind the “slow” ones.
5. During the second period $T_E/2$, the isochromats continue precessing with the same speeds as before. The fast spins catch up the slow ones and all the isochromats come back into phase exactly at $t = T_E$. The vector sum of these isochromats increases until it reaches a maximum at $t = T_E$.

Notice from Fig. 3.9 that the spin echo signal does not reach the same level as the original magnetisation. This is because there is an irreversible component to the decay.

$T_2$ decay is caused by the fluctuating magnetic fields that we talked about earlier. These are random and change with time. They are not reversible. $T_2$ affects cause the decay of each individual isochromat.
**3.7 Spin Echoes**

- $T_2'$ decay is caused by magnetic fields that vary in space, but are *constant in time*. This is what allows the effects to be reversed.

---

**Figure 3.9:** Pictorial representation of 90°–180° pulse sequence and the signal received.
4. Magnetic Resonance Imaging

4.1 Fundamentals of MRI

4.1.1 The aim of MRI

- The goal of an MRI imaging experiment is to obtain spatial information about the $^1$H nuclei in the sample. (Imaging of other nuclei, such as $^{19}$F is possible, but imaging of hydrogen is far and away the most common case.)

- Put another way, we want to map out the spatial distribution of the *proton density* function $\rho(r) = \rho(x, y, z)$. Note that the $x$, $y$ and $z$ here are co-ordinates in real space and are referred to the centre of the NMR magnet. You should distinguish these from the $x'$, $y'$, $z'$ which referred to axes centred on an individual nucleus of Section 3.3ff.

- The method of achieving this in the three different dimensions is different. We will consider three *orthogonal* directions, which, for reasons which will become clearer, we will call the *slice*, *read* and *phase* directions.

4.1.2 Magnetic Field Gradients

- The key idea in MRI is that if one makes $B$ vary with *position*, then different locations in the samples will give signals that can be distinguished from one another by their frequencies.

- Notice that, in order to change the precession frequency, it is the *field* in the $z$-direction which must change with position. This is because the nuclear spin is precessing around the $z$-axis.

![Figure 4.1: Variation of the magnetic field and precession frequency with distance. Note how $\omega$ in this case represents the *demodulated* angular frequency, i.e., the frequency in the rotating frame of reference. On the right hand figure, the principle of slice-selection is illustrated.](image-url)
4.1 FUNDAMENTALS OF MRI

It turns out to be much easier if we make the rate of change of our magnetic field constant at all points, i.e., we set up a constant magnetic field gradient. Suppose we were interested in mapping the variation of proton density in the $x$-direction:

\[
\omega = \gamma B G x \Rightarrow \partial \omega \partial x = 0
\]

[4.2]

We will find it convenient during the rest of this section to work with the demodulated angular frequency, which we will denote by $\omega$. We will arrange our demodulation so that $\omega$ is zero at the origin of the co-ordinate system. This corresponds simply to the case

[4.3]

By simply changing the direction of the gradient, we can look at the distribution of spins in different directions. Magnetic field gradients map space onto frequency.

4.1.3 Dimension 1: The Slice Direction

- Normally an MR image is a 2-D “picture” of a slice through a 3-D object.
- The great advantage of MR over X-ray CT is that we can select a slice at any position we like. This is achieved by exciting only those spins in a particular plane.
- Remember that in NMR, we see only the magnetisation precessing in the $xy$-plane. $z$-magnetisation is “invisible”.
- The slice-selection procedure consists of two components:
  (i)
The response for our normal 90° pulse can be summarised by:

\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\]

where the column vectors represent the magnetisation before and after the pulse. We then see in our MRI experiment the spins in the \( y' \)-direction, which (in the lab frame) start to precess and induce an emf in the detector.

When we use a slice selective pulse, this situation is modified to

\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\]

where \( M_x(\omega) \), \( M_y(\omega) \) and \( M_z(\omega) \) are functions of \( \omega \), which depend on the form of the slice selective pulse.

A slice selective pulse is simply a pulse whose amplitude (and possible phase) varies with time. I.e.,

\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
\]

Depending on the functional variation of \( B_1 \), slices with different frequency (and hence spatial) profiles can be selected: a given \( B_1(t) \) will give rise to a given \( M(\omega) \).

Remember: Gradients map space onto frequency. This means that a pulse which is frequency-selective becomes spatially-selective when we apply a gradient at the same time as the pulse.

**4.1.4 Dimension 2: The Read Direction**

After our imaging slice has been selected, we turn off the slice gradient and turn on a gradient in an orthogonal direction, called the read direction.
• Suppose the read direction is the x-direction. Irrespective of their y- and z-positions, spins will precess at an angular frequency

\[ \omega = -\gamma G_x, \]  

[4.7]

which depends only on the x-coordinate. Because of the way in which this process relates spatial position to (angular) frequency, it is often called \textit{frequency encoding}.

• Each different x-location will contribute to the total signal in direct proportion to the density of spins at that position:

\[ dS(t|x) dx \propto \rho(x), \]  

[4.8]

where \( \rho(x) \) is the projection of all the spins in the selected slice onto the x-axis, i.e.,

\[ \rho(x|y,z) = \lim_{\Delta y \to 0, \Delta z \to 0} \int_0^\infty \int_0^\infty \rho(x,y,z) dy dz. \]  

[4.9]

• The signal from the whole sample is simply

\[ S(t|x) \propto \int_{-\infty}^{\infty} \rho(x) \omega dx. \]  

[4.10]

In NMR, we very rarely measure the absolute value of the proton density and so usually drop constants of proportionality. Inserting the expression for \( \omega \) gives

\[ \int_{-\infty}^{\infty} \rho(x) dx \propto S(t|x). \]  

[4.11]

• You should notice that this looks very similar to a \textit{Fourier transform}. To make it into one, we perform a \textit{change of variable}, which is equivalent to a \textit{rescaling}.

Let \( k = \gamma x \) and consider \( S'(k) = S(k/\gamma x) \).

\[ S'(k) \propto \int_{-\infty}^{\infty} \rho(x) dx. \]  

[4.12]

The rescaled signal is the Fourier transform of the proton density.

\[ \uparrow \]

The proton density is the inverse FT of the NMR signal once it has been rescaled.
4.1.5 Dimension 3: Phase Encoding and the Spin-Warp Sequence

- Consider the pulse sequence in Fig. 4.4. It consists of the following elements:
  1. a 90° slice-selective pulse in conjunction with a gradient in the \( z \)-direction;
  2. a gradient along both the \( x \)- and \( y \)-direction for a period \( \tau \);
  3. acquisition during a constant gradient in the \( x \)-direction.

- How does the sequence work?
  1. The pulse selects a slice in the \( z \)-direction, as explained in Section 4.1.3.
  2. Immediately after the r.f. pulse, the only gradient is in the \emph{phase-encode direction}. This means that the spins have a precession frequency which depends on both their \( x \)- and \( y \)-coordinates.

\[
\gamma \omega \tau \gamma \phi \phi y x G x G y x y G x G y \tau y x y y G x G z T / R \]

During the period before the acquisition starts, the spins acquire a \emph{phase} which depends on their \( y \)-position:
3. During the acquisition itself, the spins are precessing, with an angular frequency that depends only on $x$.

\[ [4.16] \]

An isochromat at the point $(x, y)$ thus gives the following contribution to the total NMR signal:

\[ [4.17] \]

In the exponential term with the $x$’s, notice how the $\phi_x(x)$ is independent of time. If we make a suitable redefinition of our time variable $t' = t - t_0$, where $t_0$ represents the time taken for the read gradient to exactly reverse the dephasing caused by the read dephase gradient lobe, we can express the signal from the spins as a combination of a frequency-encoding term in $x$ and a phase-encoding term in $y$.

\[ [4.18] \]

Again, we obtain the total signal by integrating over the whole sample:

\[ [4.19] \]

4. For a single acquisition period, we obtain data at a whole range of $t'$ values, but a single $\tau$. We repeat the sequence a number of times to get data for different values of $\phi_y(y)$ by stepping the phase-encoding gradient $G_y$.

- How do we find $\rho(x, y)$?

Notice that Eq. [4.19] is very symmetrical with respect to $x$ and $y$. We can emphasise this even more by defining $k$ variables as before:

\[ [4.20] \]

Performing the change of variables as before leads to
This is a \textit{two-dimensional} Fourier transform.

\begin{array}{l}
\text{The 2-D signal is the 2-D Fourier transform of the proton density.}\\
\therefore \text{To obtain an image, we must:}\\
1. \text{Measure the NMR signal at different values of } t' \text{ and } G_x = k_x \text{ and } k_y.\\
2. \text{Perform a 2-D inverse FT on the data.}\\
\rho(x, y) = \iiint_{-\infty}^{\infty} S'(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y \quad [4.22]
\end{array}

4.1.6 \textit{Digital acquisition and image resolution}

- The above method thus gives a straightforward method for acquiring an image. Execute the imaging sequence once for every set of \((k_x, k_y)\), then arrange the data appropriately and take the Fourier transform — simple!

- The only problem is, there are an infinite number of possible values of \((k_x, k_y)\) and so it would take an infinite amount of time to acquire the image and infinite computer disk space to store it.

- The consequence of sampling the signal discretely is that \(\rho(x, y)\) is not available in its entirety. We cannot resolve objects closer together than a certain distance.

- The smallest distance apart at which we can distinguish between two separate objects is known as the \textit{resolution} of the image. The resolution may be different in different directions.

- There is a simple relation between the step length in k-space (which translates into a gradient step via formula [4.20]) and the \textit{field-of-view} of the image.
4.1 Fundamentals of MRI

If in addition, we know that there are $N_x$ samples in the $x$-direction, such that the total expanse of k-space covered is $K_x = N_x \Delta k_x$ then

Typically in medical imaging, we talk about images in terms of their pixel resolution. A low-resolution head image would have, typically, 64 pixels in each direction, whilst a high-resolution image might have 512 pixels in each direction.

4.2 Image Contrast

In practice, many biological tissues in the body have very similar values of $\rho(x, y)$. An image sensitive only to $\rho$ would not distinguish between different tissues and so would not be useful medically.

4.2.1 $T_2$ relaxation contrast

Consider the expression for the NMR signal including a term corresponding to $T_2$ relaxation. Magnetisation decays during a time $T_E$, as defined in Fig. 4.4.

$$S(k_x, k_y) = \int_{2D \text{ slice}} [\rho(x, y, z_0) e^{-T_E / T_2(x, y, z_0)}] e^{-i(k_x x + k_y y)} \, dx \, dy \, \Delta z .$$

Notice that the signal we expect is weighted by a factor $\exp[-T_E/T_2(x, y, z_0)]$ that varies spatially. When we perform the inverse FT to obtain the image, what we get out is not $\rho$, but

which is called the $T_2$-weighted proton density. Note how $T_2$ may vary spatially, i.e., the maths allows for different organs to have different values of $T_2$. 

29
Figure 4.5: Stages in the formation image: (a) Acquire a set of echoes each for a different value of gradient; (b) assemble all the data into a square matrix — a surface plot of the data is seen here and it is easy to see that by taking the data along a line through the middle, you get a plot like (a); (c) perform a 2-D Fourier transform on the data.
4.2 IMAGE CONTRAST

4.2.2 $T_1$ relaxation contrast

- Recall that to obtain an image, we need to repeat the basic sequence lots of times, each time with a different value for the phase-encoding gradient.

- $T_1$ relaxation changes the image appreciably whenever the repetition time $T_R$ between phase-encode steps is less than about 3–5 $T_1$. We state without proof that when a spin echo imaging sequence is used with a $90^\circ$ initial pulse, then the $T_1$ modification to the signal intensity is

$$\rho \propto 1 - e^{-\frac{T_E}{T_2(x,y,z_0)}} \cdot e^{-\frac{T_R}{T_1(x,y,z_0)}} \cdot \rho(x,y,z_0).$$  \[4.27\]

- This is the $T_1$-weighted proton density.

4.2.3 Summary of Image Contrast

- The image intensity $I(x,y)$ of an NMR image is

$$I(x,y) \propto [1 - e^{-\frac{T_E}{T_1(x,y,z_0)}}] \cdot e^{-\frac{T_R}{T_2(x,y,z_0)}} \cdot \rho(x,y,z_0).$$  \[4.28\]

- The observed image intensity is dependent on $\rho$, $T_1$ and $T_2$, each of which is a tissue parameter and is spatially dependent, but also on $T_E$ and $T_R$, which are experimental parameters.
By changing the imaging sequence parameters, $T_E$ and $T_R$, we can manipulate the image contrast. This is the major feature of MRI that sets it apart from other imaging modalities such as X-ray CT and ultrasound.

- NMR imaging sequences are tailored to particular pathologies.

- Examples of $T_1$ and $T_2$-weighted images are shown in Fig. 4.6.
4.3 Fast Imaging

- The time taken to acquire a typical spin- or gradient-echo image is relatively long (in excess of 2 minutes for an image with high spatial resolution).

- This leads to a number of problems, including patient discomfort, motion artifacts due to involuntary patient movement during the scan and the inability to image dynamic processes.

- As a result, a number of fast imaging techniques have been developed over the years.

- You do not need to know the details of any of these methods, but you should be familiar with the following names:

  ◊ EPI — Echo Planar Imaging, an ultra-fast but technically difficult sequence, capable of producing images in 30–100 ms

  ◊ FLASH — Fast Low-Angle Shot, a rapid sequence using low flip-angle RF pulses, giving images in 100–500 ms

  ◊ Turbo Spin-Echo — fast version of the spin-echo sequence, which acquires multiple spin echoes and which gives images in approximately 10–20 s
5 USES OF MRI

5. Uses of MRI

MRI is a method of “seeing inside things”, in particular, people. The over-riding benefit of clinical MRI is that, unlike most other imaging techniques, it is completely non-invasive. Whereas X-rays and CT involve ionising radiation, PET involves injection of a radioactive tracer and even ultrasound might potentially have some biological effect, a properly conducted MRI scan is as far as we know completely non-invasive.

5.1 Anatomical Imaging

• Anatomical imaging is what most hospital scanners are doing most of the time. In this context, “anatomical” means “visualising the body’s anatomical structure” or “distinguishing between different organs and tissues”.

• By finding out the shape, size and appearance of various structures (e.g., cancerous tumours), one can assess the pathology of a particular case.

• We distinguish this type of study from “functional” or “physiological” studies in which the operation of the organs is studied.

• Current whole body systems can obtain images with a voxel resolution of ~1mm$^3$.

• Image contrast is obtained largely as a result of relaxation time differences between the various different tissues.

5.2 NMR Parameter Mapping (QMRI)

• An important advantage of MRI is the sensitivity of the NMR signal to a number of different physical and chemical properties of the sample.

• We can use this sensitivity to make images of the parameters: $\rho$, $T_1$, $T_2$, $\sigma$, $v$ (the flow velocity of the spins) and $D$ (the self-diffusion coefficient of the spins). A number of these parameters are useful in medicine.

• While not intrinsically useful in themselves, the relaxation times $T_1$ and $T_2$ are related to a large number of other properties, such as tracer concentration, radiation dose, temperature and local blood oxygenation. By making images of $T_1$ or $T_2$, we can understand better a number of phenomena.

• By making an image which is sensitive to chemical shift $\sigma$, we can map out the spatial distribution of chemical metabolites in the brain and thus help to understand how many disease processes occur.

• If we measure the velocity $v$ of spins corresponding to blood in the arteries and veins, we can assess whether blockages have occurred. This subject is known as angiography. One of the key areas of study is coronary angiography.
• The diffusion coefficient $D$ is important because it changes greatly in brain tissue after certain types of trauma. A particular use is in detecting the regions affected by a stroke.

5.3 Intervisional Imaging

• This means imaging during an operation.

• At present, the normal course of events is for a patient to have images taken prior to the operation. The surgeon can then refer to it in planning and as he or she works. A post-operative set of images is often acquired to check how well the operation succeeded.

• The advantage of imaging as the operation is in progress is that by tracking the position of a scalpel, biopsy needle, etc., one can locate more accurately the tissue to be removed.

• A number of technical problems have had (and are still) to be overcome before interventional imaging becomes a reality. For example, you cannot use a metal scalpel in the magnetic field! Only a few hospitals in the country are capable of performing such treatment.

5.4 Functional Brain Imaging (fMRI)

• fMRI has been the hottest research topic in MRI over the last five years.

• The aim is to obtain images of brain function — we “watch” your brain in action.

• How do we do the experiment?
  1. We acquire two images: the first “at rest” and thinking of nothing, the second while performing a task.
  2. We take the difference between these images. The difference is non-zero in those areas which have changed, i.e., where something is happening.

• Why does it work?
  1. Brain activity involves a change in local blood flow.
  2. This changes the relative amounts of oxy- and deoxyhaemoglobin.
  3. The extra oxygen causes changes in the local $T_2^*$ of the region where there is extra blood flow. This leads to a change in signal.

• The effect is known as BOLD (Blood Oxygenation Level Dependent) contrast.
6. Components of an MRI System

![Block diagram of a magnetic resonance imaging system](image)

**Figure 6.1: Block diagram of a magnetic resonance imaging system**
6.1 Magnet

- This represents the largest capital cost of the MRI system (£100k — £1M depending on physical size, field strength, etc.).
- Its purpose is to produce a highly uniform field $B_0$ across the sample.
- Today, most magnets are superconducting solenoids.
- $B_0$ ranges from $<0.5 \text{ T} – 8 \text{ T}$ for whole-body horizontal-bore systems and up to $\sim15 \text{ T}$ for narrow vertical-bore microscopy and chemical systems.

6.2 Gradient Coils

- The gradient coils generate the spatially varying magnetic fields across the sample that are needed for mapping spatial position onto frequency.
- The basic design is that of a Maxwell pair of coils to create the variation in the $z$-direction and a set of saddle (Golay) coils for the $x$- and $y$-directions. See Fig. 6.2.
- The cost depends on the physical size of the gradient coils. They range from a small set 10cm long for micro-imaging, which we could make in the Department’s
workshop for under £50, up to a whole-body gradient set 2m long and weighing half a ton, which would require specialist engineering and cost up to £50000.

- The typical magnitude of the gradient produced (i.e., \((\partial B_z/\partial z)_{\text{max}}\)) is from about 20 mT/m for a whole-body system up to 1 T/m for a small-bore system.

6.3 Gradient Amplifiers

- The purpose of the gradient amplifiers is to transform an electronic signal from the controlling console in the range, say, 0–10 V into a very large current through the gradient coils.

- In MRI, we need to change the gradient very quickly, e.g., from 0 to \(G_{\text{max}}\) in 100 µs. This means changing the current through the coils very rapidly.

\[
\frac{\partial I}{\partial t} \text{ is very large.}
\]

- Now, recall that the back emf in a coil caused by changing the current through it is

\[
E = -L \frac{\partial I}{\partial t}.
\]  

[6.1]

The faster we change the current, the higher the voltage the amplifier must be capable of producing in order to overcome the back-emf. The gradient coils are designed to have the minimum possible inductance \(L\), but even so, a high voltage is needed.

- Up to 200 A is drawn from a three-phase supply at up to 200 V, giving a power of up to 40 kW. Note that this level of power is only used for very brief periods.

- Gradient amplifiers typically cost £5–10k per gradient axis

6.4 RF Coil (Probe)

- The RF coil has two purposes:
  
  (i) to transmit RF into the sample (i.e., create the oscillating \(B_1\)) to excite the nuclear spins;

  (ii) to detect the magnetic fields created by the precessing magnetisation.

- Remember: the main field \(B_0\) is along the z-axis, but the RF field is along the \(x'\)-axis, perpendicular to \(B_0\).

- If the sample can be completely contained in the probe and loaded transversely to \(B_0\), then a solenoid coil can be used — see Fig. 6.3. This gives very good signal-to-
noise. However, this is usually very inconvenient, especially for people and other coil configurations are used (e.g., saddle coil, birdcage).

- The major cost in making an RF coil is the high power capacitors which are attached to the coil to make it into a resonant circuit. Each capacitor can cost several hundred pounds and the whole cost of the coil may be in the region £200–£2000.

6.5 RF Power Amplifier
- The RF amplifier drives current through the probe sufficient to create the oscillating $B_1$ field.
- The magnitude of $B_1$ is typically (for a head probe) $\sim 2 \times 10^{-5}$ T, i.e., $\sim B_0 / 10^5$. This corresponds to a 90° pulse length of about 250 $\mu$s.
- The amplifier is often described in terms of the power broadcast. Typically $P$ is in the range 1–5 kW.
- The cost is $\sim$£1k – £10k.

6.6 Signal Amplification
- Signal amplification comes in several stages. The most important element is the pre-amplifier, which changes the signal from one in the $\mu$V range to one suitable for input to the later stages.
- Until the signal gets past the pre-amp., it is still very susceptible to the introduction of noise.
6.7 Demodulation and Phase-Sensitive Detection

- Demodulation was explained in Section 3.6.3. Its purpose is to “subtract” the frequency $\omega_L$ from the signal. This is equivalent to observing the signal in the rotating frame of reference.

- Until the signal is “mixed down”, it cannot be digitised.

- Immediately after the demodulation is a step called phase-sensitive detection. This takes the input signal and splits it into real and imaginary channels. At the end of the whole process, we end up with two voltages; these correspond to the $x'$ and $y'$-components of the magnetisation in the rotating frame.

6.8 Modulator

- Unsurprisingly, the modulator does the opposite of the demodulator. It is needed to take an RF pulse shape, which the computer provides as an “envelope”, and turn it into a $B_1$ field oscillating at the Larmor frequency.

- The modulator is essentially a multiplying device. It has two inputs, a reference frequency $\cos \omega_L t$ and the envelope function $B_1 (t)$. The output of the modulator is a voltage $B_1 (t) \cos \omega_L t$, which goes straight to RF power amplifier.

6.9 Waveform Memory, Pulse Programmer and Acquisition Computer

- These components control the MRI experiment.

- The functions of the three components are closely connected and in practice, the waveform memories and pulse programmer are normally on boards inside the computer.

- The waveform memories store the shapes of the gradient and RF pulses.

- The pulse programmer sends signals to trigger the various devices to act.

- The computer compiles the pulse program, “fills up” the waveform memories and pulse programmer, then sends the signal to start the sequence. It receives and processes the data which the instrument acquires and provides the user interface.

6.10 NMR Equipment at the University of Surrey

- 9.4 T NMR microscope and Stray Field Imaging (StraFI) system — ultra high resolution imaging for materials characterisation.

- 0.7 T small-object imaging system

- Analytical chemistry spectrometers — 4, with field strengths between 2 and 9.7T
• **Benchtop NMR system** — for non imaging analysis of relaxation and diffusion properties of samples.

• **Earth’s field NMR demonstration** — in the undergraduate lab.
7. And Finally ... Is it Safe?

- A properly conducted clinical MRI scan has no known safety risks.

- Magnetic fields in the range 0 – 2.5 T are FDA-approved for use with patients and, despite 20 years’ experience, no adverse effects on patients have been noted.

However ...

- *Metal objects* must be excluded from the vicinity of the magnet. Safety precautions are necessary to prevent:
  - ◇ ferromagnetic objects flying into the magnet — e.g., tools, scissors, oxygen cylinders (fireman’s breathing apparatus if there is a fire), etc., as well as press photographers’ cameras and vacuum cleaners!;
  - ◇ credit cards being wiped;
  - ◇ patients with implanted ferromagnetic objects being scanned — implants may move or be disturbed causing serious problems e.g., surgical clips, shrapnel, metal fragments from metal-working accidents, etc.

- *Heart pacemakers* can be disturbed by the strong magnetic field. Patients with pacemakers may not be scanned.

- There are FDA limits on the gradient switching rate — Faraday’s Law says that

\[
E \propto -\frac{\partial B}{\partial t},
\]

so rapidly switched gradients can induce unwanted EMF’s in the body. This can cause stimulation of peripheral nerves if \(\partial B/\partial t\) is too high. However, it is only a problem when pushing image acquisition speed to the limit. A normal MRI scan will not even approach the FDA limit.

- There is an FDA limit on the *RF power deposition*. This limits the number and flip angle of the RF pulses used. Because the human body is a conductive medium, it absorbs RF electromagnetic energy and dissipates it as heat.

- Some people notice temporary dizziness and nausea in fields of 3 T and above. An interesting study showed that a number of other reported effects may be psychosomatic!
II  Computed Tomography Imaging

8. Recommended material for further study

- This course is intended to provide only a very brief overview of the acquisition and usage of X-ray CT images. Further details and examples of images may be found on the Internet. A good website to start you off is:

  http://www.iwr.uni-heidelberg.de/groups/ngg/Tutorial/TutCT_121203_Lauritsch.pdf

- Some comprehensive textbooks are available at a higher level. For example:


9. CT scanning and reconstruction

9.1 What is computed tomography (CT) imaging?

- Like MRI, computed tomography is a method that can be used to create cross-sectional images.
- CT refers to a generalised methodology for image reconstruction, whose practical applications can be sub-divided into two sub-areas: emission CT and absorption CT.
- Examples of the former include positron emission tomography (PET) and single-photon emission computed tomography (SPECT). In this course, we will consider only absorption CT and, in particular, X-ray CT, as used widely in hospitals.
- The goal of X-ray CT is to reconstruct an image whose signal intensity at every point in the region imaged is proportional to $\mu(x, y, z)$, where $\mu$ is the linear attenuation coefficient for X-rays.
- Note that in practice, $\mu$ is a function of X-ray energy as well as position and this introduces a number of complications that we will not investigate here.
- X-ray CT is now a mature (though still rapidly developing) technology and a vital component of hospital diagnosis.

9.2 The first-generation CT scanner

9.2.1 Principles of X-ray absorption

- Absorption CT imaging is based on Beer’s Law, which describes how X-rays are attenuated as they pass through a medium. In a uniform substance of linear attenuation coefficient $\mu$, the X-ray “intensity”, as measured by a detector placed at depth $d$ is

  \[ I = I_0 \exp(-\mu d) \]  

  where $I_0$ is the intensity measured at depth zero.
- Suppose we now consider a set of blocks of different material, each of width $\Delta y$, as shown in Figure 9.1. The X-ray intensity measured at the exit of the set of blocks is

  \[ I(y) = \sum_{i=1}^{N} I_i \exp(-\mu_i \Delta y) \]  

  where $I_i$ is the intensity of the $i$th block and $\mu_i$ is its linear attenuation coefficient.
- For the limit $\Delta y \to 0, N \to \infty$, this becomes
9.2 The first-generation CT scanner

As shown in Figure 9.1a, the first-generation CT apparatus consists of a source and detector, placed on either side of the object to be imaged. These slide along in tandem.

Consider the intensity of the X-ray signal received by the detector when the source-detector assembly is at position $x$:

\[
\int_{-\infty}^{\infty} \exp(-\mu x) dx = \frac{1}{\mu}
\]

9.2.2 The Radon transformation

In a first-generation scanner, the source-detector track can rotate around the sample, as shown in Figure 9.2b. We will denote the “$x$-axis” along which the assembly slides when the assembly is at angle $\phi$ by $x_\phi$ and the perpendicular axis by $y_\phi$.

Clearly, we may relate our $(x_\phi, y_\phi)$ coordinates to the coordinates in the unrotated lab frame by

Figure 9.1: Schematic diagram of a first-generation CT scanner: (a) An X-ray source projects a thin “pencil” beam of X-rays through the sample, which is detected on the other side of the sample. The source and detector move in tandem along a gantry. (b) The whole gantry rotates, allowing projection data to be acquired at different angles.
(You can easily derive this using the 2-D rotation matrix for an angle $-\phi$.)

- Hence, the “projection signal” when the gantry is at angle $\phi$ is

\[
[9.5]
\]

- We define the Radon transform as

\[
[9.6]
\]

- We define a new “space”, called Radon space, in much the same way as one defines reciprocal domains in a 2-D Fourier transform. Radon space has two dimensions $x_\phi$ and $\phi$. At the general point $(x_\phi, \phi)$, we “store” the result of the projection $\lambda_\phi(x_\phi)$.

- Taking lots of projections at a complete range of $x_\phi$ and $\phi$ “fills” Radon space with data, in much the same way that we filled Fourier space with our 2-D MRI data.
9.2.3 Relationship between “real space” and Radon space

- Consider what the sinogram for a sample consisting of a single point in real (image) space will look like in Radon space. For a given angle $\phi$, all locations $x_\phi$ lead to $\lambda(\phi, x_\phi) = 0$, except the one coinciding with the projection that goes through point $(x_0, y_0)$ in real space. From Equation [9.5], this will be the projection where $x_\phi = x_0 \cos \phi + y_0 \sin \phi$.

- Thus, all points in the Radon space corresponding to the single-point object are zero, except along the track

$$\phi = \tan^{-1}(y/x), \text{ [9.8]}$$

where $R = (x^2 + y^2)^{1/2}$ and $\phi_0 = \tan^{-1}(y/x)$.

- If we have a composite object, then the filled Radon space is simply the sum of all the individual points making up the object (i.e., multiple sinusoids, with different values of $R$ and $\phi_0$). See Figure 9.3 for an illustration of this.

9.2.4 Reconstruction of CT images

- This is performed by a process known as back-projection, for which the procedure is as follows:

1. Consider one row of the sinogram, corresponding to angle $\phi$. Note how in Figure 9.3, the value of the Radon transform $\lambda_{\phi}(x_\phi)$ is represented by the grey level of the pixel. When we look at a single row (i.e., a 1-D set of data), we can draw this as a graph — see Figure 9.4(a). Figure 9.4(b) shows a typical set of such line profiles at different projection angles.
Figure 9.4: (a) Relationship of 1-D projection through the sample and row in sonogram; (b) projections at different angles correspond to different rows of the sonogram; (c) back-projection of sonogram rows to form an image. The high-intensity areas of the image correspond to the crossing points of all three back-projections of the profiles.
9.2 The First-Generation CT Scanner

2. Place the sonogram row an angle $\phi$ in real space. Then "smear it out" evenly all the way along the $y$-$\phi$-direction.

3. Repeat steps 1 and 2 for all the lines in the sonogram — see Figure 9.4(c). Where the back-projections overlap, the signal adds constructively to give high-intensity image regions.

• This is not quite the whole story. It turns out that the image that is produced by this method is blurred, as shown in Figure 9.5.

• To get exactly the right representation of the object, we need an additional mathematical "trick" called filtering. This is not explained here.

9.3 Further generations of CT scanner

• The first-generation scanner described above is capable of producing high-quality images. However, since the X-ray beam must be translated across the sample for each projection, the method is intrinsically slow.

• Many refinements have been made over the years, the main function of which is to increase dramatically the speed of data acquisition.

• Scanner using different types of radiation (e.g., fan beam) and different detection (e.g., many parallel strips of detectors) are known as different generations of X-ray CT scanner. We will not go into details here.
III Ultrasound Imaging

10. Recommended material for further study

- This course is intended to provide only the briefest introduction to the design of ultrasound scanners. Further details and examples of images may be found on the Internet. Some good websites to start you off are:

  http://dukemil.egr.duke.edu/Ultrasound/kspace/node1.htm

  http://www.cis.rit.edu/research/ultrasound/ultraintro.htm

- The following textbooks are recommended:


11.1 WHAT DO WE MEAN BY ULTRASOUND?

11. What is ultrasound imaging?
• Ultrasound imaging operates on the same principle as radar and sonar and is similar to the echo-location method bats use to navigate.
• An emitter sends out pulses of sound. These bounce of objects and the returned echoes give us information about the object.
• The fundamental equation of ultrasound is

\[
d \, c \, t = 2
\]

\[d = \text{distance of the reflecting object from the source/detector of ultrasound};\]
\[c = \text{speed of the ultrasound};\]
\[t = \text{round-trip time of the pulse, from emission to reception}.
\]

11.1 What do we mean by ultrasound?
• *Acoustic waves* with frequencies above those which can be detected by the human ear. In practice, \(20 \text{ kHz} < f < 200 \text{ MHz}\). 

\[
\lambda = \frac{c}{f} = 1.5 \text{ mm at } 1 \text{ MHz}.
\]
• Gases and liquids support only longitudinal waves; solids support transverse waves as well, but these are rapidly attenuated for non-rigid, “soft” solids.

11.2 Uses of ultrasound imaging


• Simplest form of ultrasound instrument

• Pulses of ultrasound in a thin beam are emitted from a transducer into the body and encounter interfaces between different organs.

• Some of the sound energy is reflected at each stage and some continues through to be reflected in turn by deeper organs.

• The returning pulses are detected by the transducer and the amplitude of the signal is displayed on an oscilloscope. If the time-base of the scope is constant, then the distance across the screen corresponds to the depth of the object producing the echo, according to Eq. [11.1].

• A-mode imaging gives information very quickly and involves a minimum of sophisticated apparatus

• Weakness is that this information is one-dimensional — i.e., along the line of the beam propagation.

• Nowadays, this mode has been largely superseded by B-mode (see later).

• A-mode still finds uses in ophthalmology, where the simple structure of the eye makes it relatively easy to interpret the echoes and where what is required are straightforward but accurate measurements of, for example, distance from the lens to the retina.
11.1 What do we mean by ultrasound?

- Even this very primitive instrument is not as straightforward as it might seem. To understand why, we need to look at a number of principles of physics, engineering and signal processing.

11.4 Reflection coefficients (F 21, C 26–36, W 325)

11.4.1 What causes the reflections?

- Reflections occur when the incident wave encounters a boundary between two materials with different acoustic impedances.

- Acoustic impedance \( Z \) is the material property which relates pressure changes \( p \) (in excess of atmospheric) to the vibrational velocity \( u \) of the particles in the medium.

\[
\frac{p}{Z} = \frac{u}{c}
\]  

[11.2]

If we are looking at a single plane wave through a substance with density \( \rho \) and speed of sound \( c \), then \( Z = \rho c \). See the Appendix for a more lengthy description.

- When an incident plane wave, with amplitude \( p_i \), travelling through a medium with acoustic impedance \( Z_1 \) hits a boundary with a second material of impedance \( Z_2 \) at normal incidence, there is in general both a reflected wave and a transmitted wave:

\[
\frac{p_r}{p_i} = \frac{Z_2}{Z_1} - \frac{Z_1}{Z_2} = \frac{Z_2 - Z_1}{Z_1 Z_2}
\]  

[11.3]

11.4.2 What is the significance of all the reflection coefficients?

(i) Too little reflection is bad. \( p_r / p_i \to 0 \)

Useful images occur only where there is a difference in acoustic impedance. Tissues with strikingly different properties in other respects may have similar acoustic impedances. From Fig. 4, you can see that there is virtually no reflection at a transition from liver to spleen and so the two tissues will not be delineated one from the other.

(ii) Too much reflection is bad. \( p_r / p_i \to \pm 1 \)

If the difference in acoustic impedance is too high, then virtually all the incident ultrasound will be reflected. This means that the boundary is opaque to ultrasound. The organ in question will show up very brightly, but we won't be able to see through it to find out what is underneath.
No ultrasound images of brain in vivo; skull reflects ultrasound.

Images of the heart have to be taken “round” the ribs, which are also opaque.

Finding the right “window” into the body is important.

(iii) The ultrasound transducer must be “coupled” to the body using a special gel. Before an ultrasound scan, a thin layer of gel is smeared onto the skin. Why?

**Answer:** (F p.45–47)

- The material from which transducers are made has a very different acoustic impedance $Z_{\text{transducer}}$ to that of the body $Z_{\text{tissue}}$ and more importantly that of air $Z_{\text{air}}$.

- These large “mis-matches” between $Z_{\text{transducer}}$ and $Z_{\text{tissue}}$ and between $Z_{\text{transducer}}$ and $Z_{\text{air}}$ mean that the reflection coefficients at these interfaces are close to $-1$. Little of the signal gets through gets through at a transducer-tissue boundary ($p_r/p_i \approx -0.86$) and virtually none at a transducer-air boundary ($p_r/p_i \approx -0.9997$).

- By applying the coupling gel, we exclude all air from the region between probe and body. This means that the worst case scenario of reflection from a transducer-air boundary is avoided. The reflection coefficient is still high (0.86), but imaging is possible.

- Some manufacturers use a trick called impedance matching to increase the amount of transmitted radiation through a transducer-tissue interface. Sec. A9 explains mathematically how this works. Inside the probe, there is a matching layer of thickness $\lambda/4$ between the transducer and the tissue. The acoustic impedance of the matching material is approximately:

\[ Z_{\text{match}} = 11.4 \]

- The technique has analogues in optics (“blooming” of lenses), electronics (coaxial transmission lines) and quantum mechanics (scattering of particles by potential wells). Note that this technique is not suitable in all cases and, in particular, a $\lambda/4$ layer will match completely only a single frequency of ultrasound.
11.5 What other aspects of wave propagation are important?

11.5.1 Scattering (F 22–25, C 189, W 324)

- Formulae in Sec. 11.1 strictly valid only for an infinite plane reflecting surface. In the body, there are many structures which are much smaller than this (e.g., lung tissue is a fine network of air-filled tubes). These give rise to a whole series of interfaces, at random orientations, and the reflections from these scatter the incident wave.

- At a smaller scale where \(d \ll \lambda\) (e.g., red blood cells), Rayleigh Scattering occurs and the degree of scattering varies as \(\propto 1/\lambda^4\).

- This means that low frequency ultrasound penetrates tissue better.

11.5.2 Absorption (F 23–25, C 214, W 323–324)

- A phenomenon by which organised vibrations of molecules (i.e., ultrasound) are transformed into disorganised, random motion. Acoustic energy \(\rightarrow\) Heat

- The mechanisms for this transfer include fluid viscosity, molecular excitations and chemical changes. It is difficult to measure the proportion of energy loss which occurs by scattering and the proportion lost by absorption.

- The combined effect of absorption and scattering may be written as

\[
|p_i| \ll |p_r|\]

\[
|p_i| \gg |p_r|\]

Figure 11.5: (a) A large degree of reflection occurs at the interface between the ultrasound transducer and soft tissue. (b) If the correct thickness of an appropriate material is built into the probe, much improved transmission can be obtained. Note that there is still a thin gel layer (not shown) between the “matching” layer inside the probe and the tissue. This has approximately the same acoustic impedance as the soft tissue and is used to exclude air.
This also applies to the peak oscillation velocity $u_0$ and the amplitude of displacement $a_0$ of the particles.

- Attenuation is approximately proportional to frequency, so that the depth of penetration goes down as $f$ rises.
- Instead of using amplitude, attenuation is often measured in terms of a reduction in the power density transported by the wave. Consider the units of $pu$, where $u$ is the particle vibration velocity:

$$\text{Pressure} \times \text{Velocity} = \text{N/m}^2 \times \text{m/s} = \text{(Nm)}\text{s}^{-1}/\text{m}^2 = \text{W/m}^2 = \text{Power/unit area}$$

- I.e., $pu$ represents the power being transported by the ultrasound through a unit area of the tissue normal to the direction of propagation. It is often also called the intensity of the ultrasound and is represented by the symbol $I$.

If we look at the power (intensity) attenuation, we see that

$$I = pu = \text{Re}[p_0 e^{i(\omega t - kx)}] \cdot \text{Re}[u_0 e^{i(\omega t - kx)}]$$

$$= p_0 u_0 \cos^2(\omega t - kx)$$

$$= I_0 \cos^2(\omega t - kx) .$$

Now $p_0(x) = p_0(0) e^{-\alpha x}$ and similarly for $u_0$. Hence

$$[11.7]$$

The power density transported decays twice as quickly as the vibration amplitude.

- Attenuation is often measured on a decibel scale, where

$$[11.8]$$

11.5.3 Diffraction (F 28–39, C 89–104, SW 328–333)

- Huygens’ Principle states that each point on a wavefront can be regarded as acting like a secondary source, emitting spherical wavelets. The new overall wave is found by summing the contributions from all the individual wavelets.
- In an ultrasound imager, all points on the surface of the transducer producing the ultrasound act like sources of spherical wavelets.
- When the ultrasound passes through an aperture, each point on that aperture is like a source of secondary wavelets.
- Interference between these wavelets gives rise to diffraction effects.
11.5 OTHER ASPECTS OF WAVE PROPAGATION

Diffraction becomes significant when the apparatus dimensions and objects examined become comparable with the radiation wavelength. Thus acoustic diffraction ($\lambda \sim 0.1\text{mm}$) is a much more significant effect than optical diffraction ($\lambda \sim 500\text{nm}$) for biological tissues.

Figure 11.6: Block diagram of a practical A-scanner. N.B. Not all A-mode scanners include a demodulator. The dynamic range values at each stage are approximate and refer to the power range in the signal. Take the square root (i.e., halve the dB value) for the corresponding amplitude ranges.

- Diffraction becomes significant when the apparatus dimensions and objects examined become comparable with the radiation wavelength. Thus acoustic diffraction ($\lambda \sim 0.1\text{mm}$) is a much more significant effect than optical diffraction ($\lambda \sim 500\text{nm}$) for biological tissues.

11.6 How is it all achieved in practice?


Fig. 11.6 shows the block diagram of a practical scanner. The new additions as compared with the simple diagram are concerned with the practical problems one discovers when one tries to *use* the reflected ultrasound signal.
11.6.1 Master Clock (PRF Generator)
- This synchronises the various parts of the scanner (e.g., transmitter, receiver, oscilloscope timebase) so that each is triggered to act at the correct time.
- PRF stands for pulse repetition frequency, the frequency at which clock pulses occur and at which ultrasound pulses are sent out into the sample.

11.6.2 Transmitter/Transducer/Receiver
- On the leading edge of each clock pulse, either a momentary voltage step, or a short sinusoidal burst of voltage is applied to the transducer.
- The transmitter which performs this must have a short rise time, i.e., it must be able to go from zero to its maximum voltage (100–200 V) very quickly (typically <25ns), in order to produce ultrasound pulses with very high frequency components.

11.6.3 Time Gain Compensation (TGC)

**Problem:**
- Ultrasound is attenuated as it passes through tissue.
- So, even for the same type of reflector, the signal is less for deeper objects.
- This effect is very significant. A typical $\alpha$ value is 0.15 cm$^{-1}$, so on a typical return trip of 10cm, the signal is reduced by $e^{-0.15\times10} = 0.22$ compared with reflections coming straight from the skin surface.

**Solution:**
- Amplify the later-arriving signals (i.e., the ones from deeper in the tissue) more. I.e., change the receiver gain with time to compensate for the echo attenuation.
- Achieved by making the gain of the amplifier dependent on a control voltage. This input voltage is changed by the TGC unit.
- Because of the logarithmic nature of the decrease in signal, the TGC should increase the gain a certain number of dB each ms.

**Worked Example**
An ultrasound beam propagates in uniform liver tissue with $\alpha = 0.15$ cm$^{-1}$. If the speed of sound in the tissue is $c = 1540$ m/s, what should be the rate of gain increase by the TGC?

$\alpha = 0.15$ cm$^{-1}$ is the attenuation coefficient for amplitude; the power attenuation is double this. Amplifiers are specified in terms of power, so $2\alpha = 0.3$ cm$^{-1}$ is what we want. In terms of dB, we have $-10 \log_{10} (e^{0.3}) = 1.3$ dB/cm to 2 s.f.
So we want a gain increase of 1.3 dB for each cm of travel to cancel this out. Now

\[ \frac{1540 \text{ m}}{1 \text{ s}} \equiv \frac{1 \text{ cm}}{\frac{1}{154} \text{ ms}}. \]

This means that the required TGC rate is

\[ \frac{1.3 \text{ dB}}{\frac{1}{154} \text{ ms}} \approx 200 \text{ dB / ms}. \]

Clearly, a specialised amplifier is needed.

- In practice, tissue type varies with depth and the situation is more complicated. The user is given a range of controls to vary the TGC. The rate of increase of gain (i.e., \( \frac{d^2G}{dt^2} \)) varies with time and hence depth. This is not an exact science! Notice, too, that by “tweaking” the time-gain controls to get a better image, we lose the information provided by the attenuation coefficient. By using this compensation we are “ignoring” the physics of the situation. The fact that one might not be able to see a particular boundary tells us something about the properties of that boundary.

11.6.4 Demodulator

- At the output of the compression amplifier, the echo signal mirrors that of the pulse, i.e., it oscillates at the ultrasound frequency of several MHz. The display is much easier to understand if this high frequency modulation is removed.

- Another way of describing demodulation is to say we want to change a signal oscillating at a high frequency to a lower frequency. This is what we saw in MRI.

11.7 The B-mode imager

- The commonest form of ultrasound imaging and the ultrasound equivalent of the radar pictures you see in those old war movies.
• A thin beam of ultrasound is scanned across the object and the strength of the returned echoes is displayed on the monitor.

• Notice that whilst in radar, full 360° coverage is required, in medical ultrasound, where only the body in front of the transducer is of interest, we look at a limited “pie-shaped” sector.

• A B-scan is “simply” an A-scan in which the ultrasound beam is moved and the results are displayed differently. “B” means “brightness”: the ultrasound signal changes the brightness of a spot on an oscilloscope screen instead of the amplitude of the trace in A-mode.

• What do we need to add to an A-scanner to turn it into a B-scanner? As soon as we try to turn the idea into a working system, we find a number of problems lurking! How do we display the data received? How do we make the beam sweep across the sample? What do our data mean?

• Fig. 11.9 is a block diagram of a generic B-scanner. Only three new items have been added: the co-ordinate generator, the video amplifier and the beam-steering device.

11.7.1 The Co-ordinate Generator

• This device is often also called the scan converter.

• It takes information about the instantaneous orientation of the beam and turns it into the co-ordinates of a line on the display monitor.
On simple systems, the CRT electron beam is physically scanned up and down the desired line (i.e., the co-ordinate generator acts as a variable voltage source to the 'scope x- and y-plates).

On more modern systems, the co-ordinate generator gives the memory location in which signal information is stored. The data is then displayed on a monitor by a computer program.

11.7.2 Compression and Amplifier

Even after passing through the TGC, the range of signals in the data is still large.

This is due to the range of reflector strengths in the body — see Fig. 11.4.
The compression amplifier transforms the data by some rule $V_{out} = f(V_{in})$, which reduces the dynamic range of the data (i.e., compresses the scale).

Typically, a 40–50 dB dynamic range for $I_{in}$ (i.e., the ratio $I_{in\ max}/I_{in\ min} \sim 10^4–10^5$) is transformed to an output dynamic range of 10–20 dB (10–100). Remember: take the square root of these values to get the corresponding voltage amplitude ranges.

This allows low intensity echoes to be seen on the same display as high intensity ones, i.e., strongly reflecting organ boundaries and weakly reflecting internal structure can be seen on the same image. A video monitor can display only about 256 values simultaneously.

This means that:

(i)

(ii)

11.7.3 The Beam-Steering Device

This is what distinguishes the different types of scanner.

There are various levels of distinction. The most basic is between static and real-time scanners.

11.7.1 Static B-Scanners

- The transducer is moved manually by the operator.
- The probe slides backwards and forwards over the patient, changing its angle.
- The image is built up line by line. Each time, the co-ordinates $\theta_1$, $\theta_2$ and $\theta_3$ tell the display where on the screen to show the results. See Fig. 11.8
- The advantage of the system is that the operator can choose which bits of the picture to update most often and to tailor the scanning motion to view the feature of interest from several different directions
- It is also very cheap.
11.7 THE B-MODE IMAGER

However ...

- The scans take several seconds to build up and form a complete picture. This is a problem if the object in question moves in the meantime. Static B-scanners are not suitable for imaging, for example, a beating heart.

11.7.2 Real-Time Mechanical Scanners (F83–84, C144–146)
- “Real time” scanners acquire anything from a few frames (images) per second up to several hundred. They are ideal for imaging motion.
- In a motorised scanner, the transducer is moved mechanically by a motor.
- Because of the difficulties of maintaining contact between the skin and a moving transducer, a larger probe is used, which contains the transducer “suspended” in a bath of oil, with a window to allow the pulses to leave.
- There are several different designs, as shown in Fig. 10. In all cases, the final device will depend on obvious mechanical engineering questions like:
  - How do you make a probe rock backwards and forwards very fast? Can you make it do so uniformly? How do you get leads to three transducers on a ring without everything getting tangled up when they rotate?
  - The major disadvantage of this type of device is that mechanical systems have an inherent speed limit.
  - The advantage is that there is no complicated (and expensive) electronics.

11.7.3 Electronic Steering — Transducer Arrays (F89–94, C146–158, W340–343)
- We shall not go into any detail here, but the basic principle is that a number of very small transducer are placed into a line and are then fired separately.
- By “firing” (i.e., sending out a pulse from) the transducers at different times, one can make composite wavefronts (Huygens Principle again!) which mimic that given out from one of the moving transducers above.
Electronic beam steering is potentially much faster than mechanical steering and also has the advantage that the order of sampling of the different lines is much more flexible.

All modern scanners work this way.