Correlation effects on the size of the Θ^+ Pentaquark

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The root mean square radius of the Θ^+ pentaquark is investigated based on a number of different quark cluster models both analytically and numerically. An oscillator wavefunction with a simple Gaussian form is used to produce a range of results for the uncorrelated potential in the COSMA, Lipkin/Karliner and Jaffe/Wilczek models of $\langle R^2\rangle^{1/2}$ = 0.55-0.77, 0.87-1.06 and 0.86-1.05 fm respectively. The FaCE tool is used to provide the numerical solution to the Jaffe/Wilczek model for a more realistic correlation between the cluster potentials, and produces an rms radius range of 0.99-1.06 fm.

Introduction

At their most elementary levels all matter is composed of fundamental particles, quarks and electrons. The quarks combine and interact via gluons, the mediators of the strong force. This interaction is responsible for holding nuclei together and allowing fusion to occur in stars, hence allowing the production of heavy nuclei and life itself. Quantum chromodynamics (QCD) is the underlying theory of the strong force and is a quantum field theory that successfully embodies both special relativity and quantum mechanics [1]. However, its equations are complicated, making it difficult to calculate the masses and properties of familiar particles from first principles. There are, however, various simplifications and models which are either QCD-based or incorporate its main principles to explain the spectrum of hadrons observed. Perhaps the most successful is the quark model, which is based on the principle that all hadrons are either composed of quark anti-quark pairs (mesons) or three quarks (baryons) [2]. There are also six quark flavours; up, down, strange, charm, top and bottom, which can have one of three colour states; red, green or blue, combining to make a 'neutral' (or 'colourless') state. The type of meson or baryon depends on which quark flavours the particle consists of. Any state with a quark content other than this is beyond the quark model and termed as exotic, although the existence of such exotic states have been proposed since the early days of QCD.

One of the most exciting developments in particle physics over the past few years has been the experimental evidence for the existence of a particle consisting of five quarks, named the pentaquark, Θ^+[3]. The decay into a proton (uud) and kaon (u$\bar{s}$) provides evidence for the minimum composition of uudd$\bar{s}$, with one unit of positive charge, positive strangeness and a baryon number of one [4]. The mass is found to be 1540 MeV, although other properties of the particle including angular momentum and parity remain undetermined. Since then, the existence of other similar exotic states have been claimed by many research groups around the world, but as yet none have been conclusively identified. It was originally thought that particles such as these would be highly unstable and decay into two lighter particles with a very short lifetime so would effectively be unobservable. However, the lifetime, or width of the pentaquark from experimental evidence was claimed to be ~20 MeV, which is 100 times lower than the expected width of a conventional baryon decaying via the strong interaction [5]. It has been suggested that the extra binding energy required to give this particle its long lifetime could be due to the quarks forming clusters inside the Θ^+. Among the many theoretical models for the exotic particle are the three-cluster state proposed by Robert Jaffe and Frank Wilczek [6] and the diquark-triquark model put forward by Harry Lipkin and Marek Karliner [7]. While the jury is still out on the existence of the Θ^+ and other states as real particles, theoretical work is necessary for understanding their fundamental properties and the way in which they form. This work could provide exciting new insights into the subtleties of the strong interaction.

The main quark cluster models are briefly reviewed here, and the size, or root mean square (rms) radius of the Θ^+ pentaquark is calculated according to each model by using the known properties from experimental evidence and considering the interaction between each of the quarks and clusters. An analytical approach assuming uncorrelated quark and cluster potentials is first adopted before using the more realistic correlated potential approach to numerically solve the Schrödinger equation for the Jaffe/Wilczek model.

Results and Discussion

First, lets consider a single particle model. This model assumes all the constituent quarks are independent of each other (i.e. decoupled) and is analogous to the Cluster Orbital Shell Model Approximation (COSMA), which is commonly used in nuclear theory (figure 1). The overall wavefunction, ψ, is assumed to be the product of the wavefunctions for each constituent with respect to the anti-strange quark. Each pair of quarks is assumed to be in a relative s-wave (i.e. angular momentum, L=0). This state is allowed according to the Pauli Exclusion Principle if each of the two like-quarks have opposing spin. The pairs are given oscillator wavefunctions, represented by a simple Gaussian function, where α is the 'oscillator parameter' defining how tightly bound each quark is to the $\bar{s}$.

$$\psi(r) = A\exp\left(-\frac{r^2}{2\alpha^2}\right) \qquad (1)$$

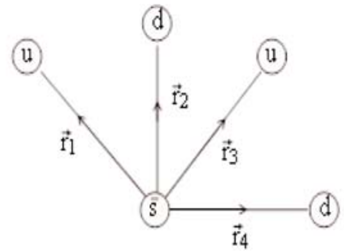

Figure 1: Quark arrangement in the COSMA Model

The normalisation condition is applied, and a standard integral used to obtain:

$$A^2 = \frac{1}{\pi\sqrt{\pi}\alpha^3}. \qquad (2)$$

The mean square radius for this method is defined as [8]:

$$\langle r^2\rangle = \frac{4m}{M}\left(1-\frac{m}{M}\right)\langle\Psi|r_i^2|\Psi\rangle. \qquad (3)$$

In this expression, m represents the masses of the u and d quarks (these are taken to be equal at 360 MeV [7]) and M is the total mass of the particle ($4m_{u,d}+ m_s$ =1980 MeV). This value is used throughout the calculations instead of the measured mass of 1540 MeV since it is not known how the binding energy is distributed within the particle. The total wavefunction is expressed in terms of its individual components, where all but the first component, $\phi(r_1)$, satisfy the normalisation condition. Standard integrals are applied to simplify the formula to:

$$\langle r_1^2\rangle = \frac{4m}{M}\left(1-\frac{m}{M}\right)\left(3/2\alpha^2\right) \qquad (4)$$

The oscillator parameter is chosen from the typical range of hadronic interactions, where the separation ranges between 0.5-1 fm [10]. This produces a range for α^2 of 0.33-0.67 fm and a root mean square radius ranging between 0.55-0.77 fm.

The Karliner/Lipkin model suggests the structure for the pentaquark is a diquark coupled to a triquark in a relative p-wave (L=1) (figure 2) [7]. Although a number of properties have yet to be experimentally determined, Karliner and Lipkin predict this state to have a spin parity of $J^P = 1/2^+$ and isospin, I=0, which produces a mass that is close to the experimental value.

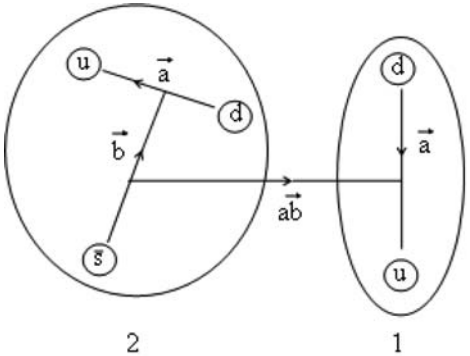

Figure 2: Quark arrangement in the Karliner/Lipkin Model

The radius of the particle for this model is then given simply by the radius of each of the two clusters, and the separation between them, with some weighting factor of the masses, where m_1 and m_2 are the masses of each cluster and the triquark cluster is classed as two separate clusters itself with a ud coupled to a point-like $\bar{s}$:

$$M\langle R^2\rangle = m_1\langle r_1^2\rangle + m_2\langle r_2^2\rangle + \frac{m_1 m_2}{M}\langle r_{12}^2\rangle. \quad (5)$$

The oscillator wavefunction for a relative p-wave state differs from that of an s-wave (eqn 1) by a factor of r. This leads to a $^5/_4\alpha^2$ dependence on the cluster separation. Since this relative angular momentum is between the two main clusters, the dependence on separation remains as $^3/_2\alpha^2$ within each cluster. The separation between the two clusters and the ud-$\bar{s}$ is held constant at 1fm and the u-d separation varied between 0.5-1 fm to provide an upper range estimate for the size of 0.87-1.06 fm. It would be expected that the $\bar{s}$ quark bound in the triquark cluster experiences a stronger potential than between the two clusters themselves, so based on the principle that the lighter the quark, the tighter they are bound a more realistic approach might be to hold the separation of the ud-$\bar{s}$ pairs constant at 0.5 fm, keep the clusters separated at 1fm and vary the ud-$\bar{s}$ separation between 0.5-1 fm. This produces a radius range of 0.82-0.87 fm.

Jaffe and Wilczek suggest that the Θ^+ can be considered as a bound state of an anti-quark with two highly correlated spin-zero diquarks (Figure 3) [6]. In this model, the lighter the quarks, the stronger the correlation, which helps the formation of the two diquarks, where they obey Bose statistics. They propose a state with overall positive parity, where only the s-wave configuration is likely (Figure 3). The rms radius is calculated from the same formula (eqn 5) where the simple s-wave Gaussian wavefunction is applied. In this case the separation of the two diquarks and that of the to the diquark pair is held constant at 1fm, while the separation of the u and d in each pair is varied to produce a size range of 0.86-1.05 fm.

Since the Jaffe/Wilczek model represents the pentaquark as a three-body problem, it is possible to further investigate it using a tool called FaCE [9] (which stands for Faddeev Calculations with Core Excitation). FaCE is a Fortran programme, which was originally designed to numerically solve three-body problems in nuclear physics, particularly halo nuclei. So far, the methods discussed for dealing with the pentaquark

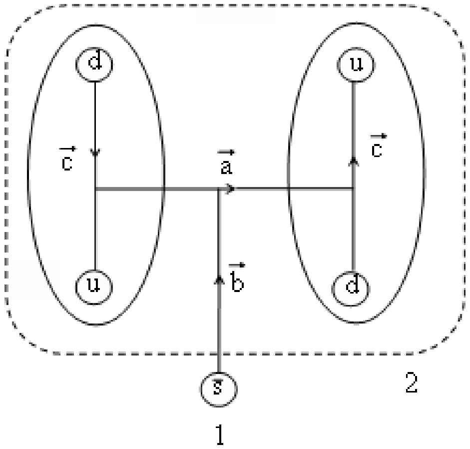

Figure 3: Quark arrangement in the Jaffe/Wilczek Model

have assumed completely independent potentials between the quarks and clusters in each model, however in a more realistic situation, the separation of each of the clusters will depend on the potentials between all of the constituents. The FaCE tool calculates energies and eigenvalues, including the rms radius for nuclei by solving the three-body Faddeev equations. This is modified to allow the option of simple quark-quark potentials in the Θ^+ particle.
An input file is read in containing information about the pentaquark such as the mass, charge, spin and parity of the clusters and the particle itself. Where these parameters are unknown, the predictions of Jaffe and Wilczek are used. The type of potential between the quarks is then selected. Initially, a simple confinement potential is used, with a linear form, where V_0 and C are the confinement parameters:

$$V_{conf} = V_0 + Cr \quad (6)$$

A first estimate for these two free confinement parameters is taken from the results of Glozman et al.[11], where they were determined from a fit to the baryon spectra in their chiral constituent-quark model. These values are V_0=416.0 and C=2.33 respectively. They produce a ground state energy of -1181 MeV and an rms radius of 1.65 fm. The energy of the ground state is the equivalent of the binding energy, where the negative result indicates a bound state. The expected ground state energy would therefore have a value of $M_{\Theta+}-4m_{u,d}-m_s$ = 440 MeV. However, the two parameters were chosen out of a possible set that lead to a similar description of baryon spectra, so it is possible to adjust them to produce a more sensible ground state energy. Values of V_0=320 MeV and C=100 lead to a binding energy of 440 MeV and rms radius of 1.11 fm.

A more realistic potential between the quarks is the linear harmonic oscillator, which was used for the analytical solutions to the pentaquark models. The harmonic oscillator potential is used for the study of particles with mass m, which are attracted to a fixed centre by a force proportional to the displacement from the centre [12]. This potential is included in the Hamiltonian operator, where the Schrödinger energy eigenvalue equation has the solution:

$$\Psi(r) = \left(\frac{1}{a\sqrt{\pi}}\right)^{\frac{3}{2}} \exp\left(-\frac{r^2}{2a^2}\right). \quad (7)$$

The parameter a has the units fm, and is related to the force constant, k, specified in the pentaquark input file in terms of the reduced mass, μ, of each cluster by

$$k = \frac{(\hbar c)^2}{\mu c^2 a^4}. \quad (8)$$

Table 1: Analytical solutions for decoupled quark cluster potentials

u-d separation [fm]	α^2	$\langle R^2 \rangle^{1/2}_{COSMA}$ [fm]	$\langle R^2 \rangle^{1/2}_{K/L}$ [fm]	$\langle R^2 \rangle^{1/2}_{J/W}$ [fm]
0.5	0.33	0.55	0.87	0.86
0.6	0.40	0.60	0.91	0.90
0.7	0.47	0.65	0.95	0.94
0.8	0.53	0.69	0.98	0.98
0.9	0.60	0.73	1.02	1.02
1.0	0.67	0.77	1.06	1.05

Table 2: Results from the FaCE calculations in a correlated oscillator potential

u-d separation [fm]	k	$\langle R^2 \rangle^{1/2}_{FaCE}$ [fm]	Energy, E [MeV]
0.5	1728	0.55	0.87
0.6	833	0.60	0.91
0.7	450	0.65	0.95
0.8	264	0.69	0.98
0.9	165	0.73	1.02
1.0	108	0.77	1.06

The reduced mass, μ, used for each diquark cluster is 360 MeV and the ud-s̄ cluster is 308 MeV, where is quoted as 197.3 MeV [12]. For a u-d separation range of 0.5-1 fm a size range of 0.99-1.06 fm is obtained. The binding energy range is 898-441 MeV. These results are summarised in tables 1 and 2.

Conclusion

In summary, each of the model calculations produced a range of possible values for the rms radius of the Θ^+ pentaquark. The Lipkin/Karliner, Jaffe/Wilczek and FaCE models provided a range estimate that was ~40% than that of the COSMA predictions, where the Lipkin/Karliner and Jaffe/Wilczek models produced results that fell within 1% of each other. The analytical solution provided by the FaCE model can be directly compared to the Jaffe/Wilczek calculation to gain some insight into the effect of the correlated potentials. At smaller u-d separations, the FaCE model predicts a larger rms radius than the Jaffe/Wilczek calculation, with being 14% larger at a separation of 0.5 fm. This is a significant difference on the particle scale. As the u-d separation increases, FaCE predicts a slower increase in the rms radius, with both models predicting a value of 1.06 fm at a 1 fm u-d separation. The slower increase is expected, because as the separation of one cluster changes, the others will also adjust in the correlated potential. Finally, a further indication for the size of the pentaquark is the energy values that are output by the FaCE model. Since the mass of the constituent quarks is 1980 MeV, but the observed mass of the Θ^+ is around 1540 MeV, approximately 440 MeV must be taken up as binding energy. FaCE predicts energies which are very close to this value for the higher u-d separations, which could provide validation for the calculated radii.

If correlated quark potentials are included in the three-body Jaffe/Wilczek model the size of the Θ^+ pentaquark is predicted to be larger than in the uncorrelated case, provided each (ud) diquark is tightly bound. However, the correlation has little effect if the (ud) diquarks are weakly bound.

The existence of the Θ^+ pentaquark is still a debated topic. Although a number of research groups have confirmed the original findings of the Θ^+, many other experimental groups have claimed negative results [13,14]. Despite this, theoretical work on this particle and other exotic states is necessary for a greater understanding of the strong interaction.

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