Notes on Accuracy: To help with coursework of Biological Physics (PHY3040)

Richard Sear

These notes were written in response to questions from students asking for help in judging the accuracy of calculations they were doing for the tutorial questions. It can be quite hard to get a good idea of how accurate a calculation needs to be, but this is a useful skill. It can be a waste of time to do a calcuation to an accuracy greater than you need. If you just need a rough estimate, they calculating to five significant figures is a waste of your time. By quite hard I mean that estimatint this is often harder than doing the calculation itself, especially if you know the formula and are just plugging in numbers.

It is also important to be able to determine how accurate is a calculation that you have done. So I also cover those.

HOW ACCURATE DOES A CALCULATION NEED TO BE?

Here, via a couple of simple examples, I try to answer the question: How do I find out how accurate a calculation needs to be. This is a different idea from how accurate a calculation is. Here we ask ourselves what we want the calculation to do, and then estimate how accurate it needs to be, to do this.

Maybe the easiest way to learn about this is via simple examples. So here are two example calculations:

1. The Péclet number, Pe, that is the ratio for the timescale for diffusion over a distance L, and the timescale for flow at speed u over the same distance. This is

$$\mathrm{Pe} = \frac{uL}{D}$$

   for D the diffusion constant.

   If you just want to know whether diffusion or flow is faster, or if they are comparable, all you really need to know is if Pe$\ll 1$, or Pe $\gg 1$, or Pe ~ 1.

   If your estimate is Pe $\sim 10^5$ then even a large error will then not make a difference. Even if your Pe could be out by a factor of 100, then it is still somewhere in the range 10^3 to 10^7. Both these values are still $\gg 1$, so they still give you the same answer: diffusion is much slower than flow.

2. For EM waves in a vacuum, the wavelength lambda is related to the energy E by

$$\lambda = hc/E$$

   for h Planck's constant and c the speed of light. Say, for example, you are looking for a sharp absorption peak in the visible range. The absorption peak has a width of only a few nm, where photons are absorbed. Then you need an accurate answer.

   An estimate $\lambda \sim 500$ nm (accurate to one significant figure) is then not good enough. You need $\lambda = 523$ nm (accurate to three significant figures).

HOW ACCURATE IS A CALCULATION THAT YOU HAVE DONE?

Here, via a couple of simple examples, I try to answer the question: How can I find out how accurate is a calculation that I have done.

1. The Peclet number, Pe, that is the ratio for the timescale for diffusion over a distance L, and the timescale for flow at speed u over the same distance. This is

$$\mathrm{Pe} = \frac{uL}{D}$$

   for D the diffusion constant.

   If you are given the values $D = 4 \times 10^{-10}$ m^2/s, $L = 0.0012$ m, and $u = 0.0192981$ m/s (the u example is deliberately silly to make a point), then as D is the least accurate number (it is given to only one significant figure), it limits the accuracy. So for most purposes you can, for example, work with $u = 0.019$ m/s or 0.02 m/s.

So, here you can estimate Pe $\sim 5 \times 10^4$

Also, the accuracy of an estimate is not only limited by the accuracy of the input values, but by the accuracy of the expression. Dimensionless numbers like Pe just compare two timescales, and so are intrinsically not-very-accurate, i.e., only accurate to one significant figure at most.

For example, both Pe = 0.5 and Pe = 1.6 mean that the timescales for transport via diffusion and flow, are comparable – of the same of magnitude. Also, if you just want to know which process is faster, as in this example, Pe is so far from 1, then approximating Pe by 10^5 would also work (on a logscale 5×10^4 is closer to to 10^5 than it is to 10^4).

2. For EM waves in a vacuum, the wavelength λ is related to the energy E by

$$\lambda = hc/E$$

for h Planck's constant and c the speed of light. This expression is essentially exact (i.e., very very accurate), and we know the values of h and c very very accurately. So if you are given a value for the energy accurate to 4 significant figures, you can (and often should) calculate λ to 4 significant figures - as that is the true accuracy, limited only by the accuracy of the energy you are given.

The simple rule of thumb is that if you are putting several numbers into an expression, the accuracy of the result is that of the least accurate part of the calculation. This is either the least accurate of the numbers, or if the expression itself is less accurate than these, then the accuracy will be that of the expression.