LECTURE 1 of Biological Physics (PHY3040)

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FERMI ESTIMATES/ ORDER-OF-MAGNITUDE ESTIMATES

As physicists we want to be quantitative and to develop models to understand what we are interested in, here living organisms. But living organisms are variable and complex, and in the biological physics part of the course we will mostly be working with rough estimates, that are only accurate to an order of magnitude. For example, I will typically take 100 kg as the mass of a human, most of us are less than that, but almost all adults are in the range 40 to 120 kg, and for a simple estimate 100 kg is good enough.

This method of working is particularly associated with the physicist Enrico Fermi, and the results are sometimes known as Fermi estimates. The calculations are also often called back-of-the-envelope calculations.

This means, for example, that if some mass is about 10^3kg, that both 500 kg and 2000 kg are right (and so would get full marks in an exam), but that the answer 1342 kg is not correct as it gives a misleading picture of the accuracy of the answer. You should always think about what level of accuracy you are working at.

Useful Numbers

	Human	Elephant	Mouse
height	1 m	3 m	5 cm
mass	10^2kg	3×10^3kg	10 g

$$\text{stress in femurs (standing)} \sim 10^6\text{Pa}$$
$$g \simeq 10 \text{ m/s}^2$$
$$\text{mass density } \rho \sim 10^3\text{kg/m}^3$$

(A)

(B)

FIG. 1. Images (from Wikimedia) of: (A) an elephant, and (B) a mouse (species = plains pocket mouse). The elephant's body is maybe 3 m long, its legs are approximately 40 cm thick, and its mass is of order 10^3kg. The mouse is the plains pocket mouse, which is about 5 cm long (excluding tail), and its mass is of order 10 g.

HOW THICK DO OUR FEMURS (THE BONES IN OUR THIGHS) NEED TO BE TO SUPPORT OUR WEIGHT?

By definition stress (=force/area) in femurs when animal is standing is

$$\text{stress} = \frac{\text{force of gravity}}{\text{total bone cross-section}}$$

$$\text{stress} = \frac{\text{mass} \times g}{\text{number of legs} \times \text{femur cross-section}}$$

For a four-legged animal that has an approximately cubic body h across, has legs of width w, and where the cross-section of a femur is 10% of the cross-section of a leg

$$\text{stress} = \frac{\rho h^3 g}{4 \times 0.1 w^2} = \frac{10^5 h^3}{4w^2}$$

as leg cross-section $\sim w^2$ so femur cross-section $\sim 0.1w^2$. Mass of animal of size h across is mass density ρ multiplied by the volume h^3.

We assume when standing that the stress in our femurs is roughly 10^6Pa (Pa = N/m^2), becuase bone is strong enough to support a stress of about 10^7Pa before breaking, so 10^6Pa when standing gives a safety factor of about ten. When we walk, run or jump the stresses are higher than when standing. So,

$$10^6 = \frac{10^5 h^3}{4w^2}$$

rearranging to get an equation for leg width w

$$w^2 \sim \frac{1}{40} h^3 \quad h \text{ in m} \tag{1}$$

If we put in our height of about 1 m, we get $w \sim 20$ cm. This is a bit large but of the right order of magnitude, 10 cm would be a better approximation for the width of typical human leg, but this calculation is very approximate, so being within a factor of two of the right answer is OK.

HOW BIG CAN THE BIGGEST LAND ANIMAL BE?

We can rewrite equation (1) to give as an expression for the ratio between the width of an animal's legs w and the width of its body h

$$\frac{\text{width of leg}}{\text{width of body}} = \frac{w}{h} \sim \frac{1}{6} h^{1/2} \quad h \text{ in m}$$

This equation says that the relative width of the legs to the body scales as the square root of the size of an organism, i.e., small organisms (eg, mice) have relatively spindly legs while large organisms (eg, elephants, large dinosaurs) have fat legs. See Figure 1.

The legs of a four-legged animal can't be more than half the width of the animal: $w/h \leq 1/2$, so left-hand side of that equation can be 1/2 at most, but right-hand side of equation increases with h. When the right-hand side reaches 1/2 then this sets tbe largest size an animal can be (on land). This occurs at $h \sim 10$ m.

Animals 100 m across cannot walk the Earth as even with very fat legs the stress in their femurs due to gravity is too great. This limit comes from simple geometry: mass $\propto h^3$, but at fixed w/h the cross-section area of the femurs supporting this weight only increases as h^2 — a slower increase with h.