7/3/01

ISTR

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Solutions to Problem sheet 1

1) Let velocity of S' rel. to S be $\underline{v}$. It has x, y, z components v_x, v_y, v_z

$\therefore$ Galilean trans. eqns. are

$$\begin{cases} x' = x - v_x t \\ y' = y - v_y t \\ z' = z - v_z t \\ t' = t \end{cases}$$

2) K.E. $= \frac{1}{2}mv^2$, where v is internal velocity of nucleons in nucleus relative to each other. This does not change when CM of whole nucleus is moving at different velocities, which is what would be seen from different frames.

3) a) Before collision, momentum of system is

$$p = m_1 u_1 + m_2 u_2 = 3 \times 4 + 1 \times (-3)$$
$$= 9 \text{ kg.m/s}$$

Since mom. must be conserved, mom. after $\bar{p}$ must be same as p

$$\Rightarrow \quad \bar{p} = m_1 \bar{u}_1 + m_2 \bar{u}_2$$
$$\Rightarrow \quad 9 = 3\bar{u}_1 + 1 \times 3$$
$$\Rightarrow \quad \bar{u}_1 = \frac{9-3}{3} = \underline{2 \text{ m/s}}$$

b) Using Galilean velocity transformation equations can work out the four velocities $u_1', u_2', \bar{u}_1', \bar{u}_2'$ in S' frame:

$$u_1' = u_1 - v = 4 - 2 = 2 \text{ m/s}$$
$$u_2' = u_2 - v = -3 - 2 = -5 \text{ m/s}$$
$$\bar{u}_1' = \bar{u}_1 - v = 2 - 2 = 0$$
$$\bar{u}_2' = \bar{u}_2 - v = 3 - 2 = 1 \text{ m/s}$$

In S' mom. before collision is
$$p' = m_1 u_1' + m_2 u_2'$$
$$= 3 \times 2 + 1 \times (-5) = 6 - 5 = \underline{1 \text{ kgm/s}}$$
After collision
$$\bar{p}' = m_1 \bar{u}_1' + m_2 \bar{u}_2' = 3 \times 0 + 1 \times 1 = \underline{1 \text{ kgm/s}}.$$
So $p' = \bar{p}' \Rightarrow$ momentum is conserved in S' even though total momentum of system is different.

4) $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

Use Binomial expansion

$$\gamma = 1 - \frac{1}{2}\left(-\frac{v^2}{c^2}\right) - \frac{1}{2}\left(-\frac{3}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \cdots$$
$$= 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \cdots$$

If $v \ll c$ then $\frac{v^4}{c^4} \ll \frac{v^2}{c^2}$

$$\therefore \gamma \simeq \underline{\underline{1 + \frac{v^2}{2c^2}}} \quad \text{(Binomial approx.)}$$