Solutions to LSTR Mock Paper

Q1 – (d)

Q2 – (c)

$\gamma = 4$

$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 4 \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{16}$

$\Rightarrow \frac{v^2}{c^2} = \frac{15}{16} \Rightarrow v = \frac{\sqrt{15}}{4} c = 0.968\,c$

$= 96.8\%$ of c.

Q3 – (a)

$L = 0.96\,L'$

$L = \frac{1}{\gamma} L' \Rightarrow \gamma = \frac{1}{0.96} = 1.041\dot{6}$

$\Rightarrow \frac{1}{1-v^2/c^2} = (1.041\dot{6})^2$

$\Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{(1.041\dot{6})^2} = 0.078$

$\Rightarrow v = \sqrt{0.078}\, c = 0.28c$.

Q4 – (d)

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = 1.\dot{6}$.

$L = \frac{1}{\gamma} L'$

$4 = \frac{1}{1.\dot{6}} L' \Rightarrow L' = 1.\dot{6} \times 4 = 6.\dot{6}$ LY

Q5 – (a)

$$\Delta t' = \gamma\left(\Delta t - v_0 \Delta x / c^2\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.6^2}} = \underline{1.25}$$

$$\Delta t' = 1.25\left(1 - \frac{0.6 \times 4 \times 10^8}{c^2}\right)$$

$$= 1.25\left(1 - \frac{0.6 \times 4 \times 10^8}{3 \times 10^8}\right)$$

$$= 1.25(1 - 0.8) = 1.25 \times 0.2$$

$$= \underline{0.25 \text{ secs}}$$

(in same order as same sign)

Q6 – (e)

$$f = \sqrt{\frac{c - v}{c + v}}\, f_0 = \sqrt{\frac{c - 0.9c}{c + 0.9c}}\, f_0 = \sqrt{\frac{0.1}{1.9}}\, f_0$$

$$= \underline{0.23\, f_0}$$

Q7 – (b)

$$\text{error} = \frac{P_{rel} - P_{non\,rel}}{P_{rel}} = \frac{\gamma m_0 v - m_0 v}{\gamma m_0 v} = \frac{\gamma - 1}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1 - 0.5^2}} = \frac{1}{\sqrt{0.75}} = \frac{2}{\sqrt{3}}$$

$$\therefore \text{error} = \frac{\frac{2}{\sqrt{3}} - 1}{\frac{2}{\sqrt{3}}} = \frac{2 - \sqrt{3}}{2} = 0.134$$

$\Rightarrow$ error of $\underline{13.4\%}$.

Q8 – (b)

$(\gamma - 1) M_p c^2 = 6 M_p c^2$

$\Rightarrow \gamma = 7$

$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{49} \quad \Rightarrow v = \sqrt{\frac{48}{49}}\, c = \underline{0.9897c}$.

Q9 – (e)

$(\gamma - 1) M_0 c^2 = \frac{1}{2} \gamma M_0 c^2$

$\Rightarrow \frac{1}{2}\gamma = 1 \Rightarrow \gamma = 2$

$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \quad \Rightarrow v = \underline{\frac{\sqrt{3}}{2} c}$.

Q10 – (c)

Assum one rocket is the moving S' frame and it is the speed of the other rocket as seen in this frame that we need.

Let $v_0 = 0.8c$ (the speed of S' rel. to S)

$\therefore v_{2c} = 0.9c$ (speed of second rocket in S frame)

$\therefore v_x'$ is speed of second rocket in S' frame (ie as seen from 1st rocket).

$$\therefore v_x' = \frac{0.8c + 0.9c}{1 + \frac{0.8c \times 0.9c}{c^2}} = \frac{1.7c}{1.72} = \underline{0.988c}$$