Faster than light?

The Great Betrayal:

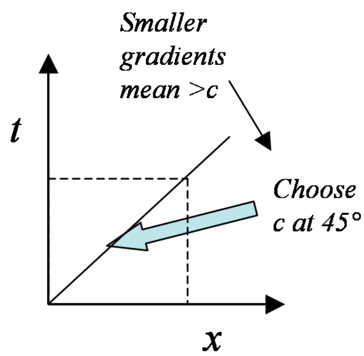
This is an example taken from the book by Taylor and Wheeler (*Spacetime).*

The Humans and the Klingons sign a peace treaty known as the Treaty of Shalimar on the Klingon's home planet Kronos. The treaty takes place four years before an event that becomes known as the Great Betrayal. It is so important that it is taken to be the origin of spacetime in all inertial reference frames. With the Treaty of Shalimar the Klingons promise not to attack the Federation fleet or space colonies. In return, the Federation allows them access to its technology.

After the treaty is signed the Federation negotiators leave Kronos in their ship and travel away in the positive *x*-direction at a speed of 0.6*c*.

For this story we need to draw spacetime diagrams. For simplicity we drop the two unused space dimensions in the y and z directions.

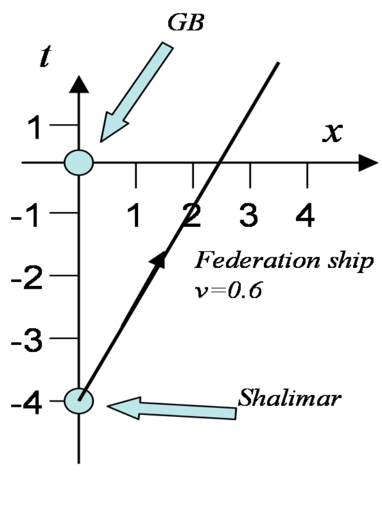
We choose the axes scales to be in units of 1 LY for x and 1 year for t. In these units, *c=1*, which is very convenient as it means we don't have to carry it around.

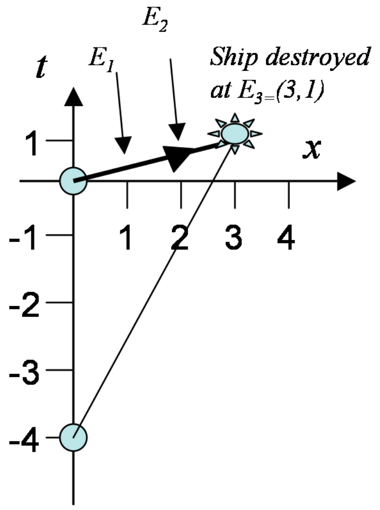


We now have to consider two different reference frames: the rest frame of Kronos and the rest frame of the Federation starship.

1. Rest frame of Kronos, *S*:

Four years after the Treaty of Shalimar, the Klingons use the technology given to them to build a faster than light (ftl) missile and fire it at a speed of 3*c* towards the departing Federation ship. This launch is what marks the Great Betrayal (GB).
Two space colonies at 1LY and 2LY away from Kronos see the missile streak past them. In this frame they are at rest and the events of the missile





going past them defined as E_1 and E_2. Of course it is travelling too fast for them to have time to warn the Fed ship. The missile destroys the ship at E_3. This event is 3LYs away from Kronos but only one year after the GB.

So far so good (well, not so good for those on board the Federation ship!

We next have to consider events in the rest frame of the Fed ship, that is the frame, *S'*, that is moving at velocity 0.6 relative to *S.*

We need to recalculate the coordinates of the different events using the LT equations. The origin of this new frame also coincides with the GB (since that is how defined this event to start with).

Shalimar treaty coordinates:

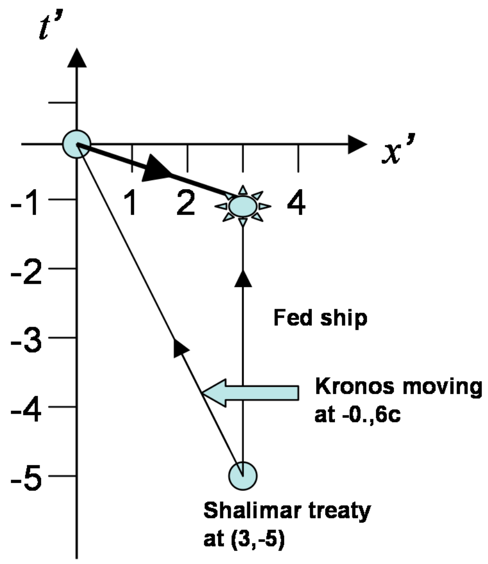
$$\gamma = 1\Big/\sqrt{1-0.6^2} = 1.25$$

$$x'_{sh} = 1.25(x_{sh} - 0.6t_{sh}) = 1.25(0 - 0.6\times -4) = 3LY$$
$$t'_{sh} = 1.25(t_{sh} - 0.6x_{sh}) = 1.25(-4 - 0) = -5Y$$

Ship destroyed at E_3:

$$x'_3 = 1.25(x_3 - 0.6t_3) = 1.25(3 - 0.6\times 1) = 3LY$$
$$t'_3 = 1.25(t_3 - 0.6x_3) = 1.25(1 - 0.6\times 3) = -1Y$$

Note that $x'_{sh} = x_{sh} = 3$ since Fed ship is stationary in its own rest frame.



So we can now draw a new spacetime diagram for the moving frame

In this frame, the Federation ship is stationary and Kronos is moving away from it in the –ve *x*-direction. Four years after the treaty is signed, the ship is destroyed at (3,-1) and, as we move up in time, the missile is reconstituted from the explosion and travels backwards, first to event E_2 then at a later time E_1 until finally landi ng back on Kronos one year *after* it has destroyed the ship!

In other words, if we track the missile more sensibly with its launch to begin with and then it destroying the ship we see that it would have to be moving back in time.
So, in this frame it would appear that the effect takes place before the cause. This is what is known as a violation of causality, which is something that is forbidden. And since no reference frame is any more valid than another, then the only way to avoid such a paradox is to rule out faster than light travel.

Further evidence of *c* being the maximum speed in the Universe:

1. What happens to time when travelling at s.o.l.?

Clock moving at speed *v* will appear dilated by $\Delta t = \gamma \Delta t'$. Therefore if $v = c$, $\gamma = \infty$

$\therefore \Delta t = \infty$

i.e. interval takes infinitely long time. That is time stands still (eternal moment).

2. What happens to time at $v > c$?

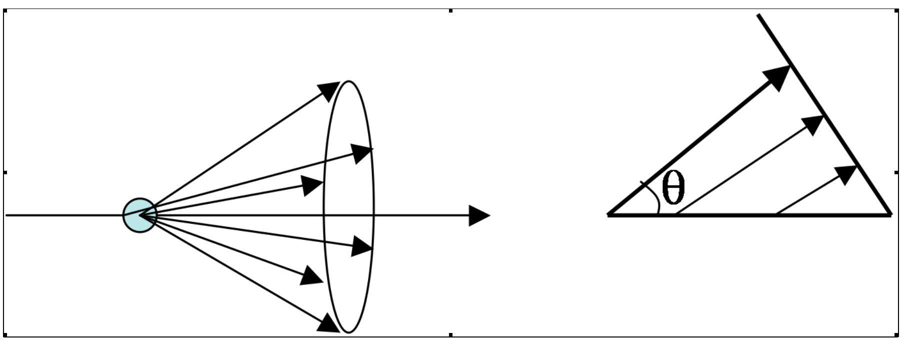
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{-\text{ve}}} = \text{imaginary number}.$$

$\therefore \Delta t =$ imaginary time!

So, L.T. equations tell us that it makes no sense to think about ftl travel It has been hypothesised that ftl particles might exist, known as 'tachyons'. But their properties would be so weird that their existence is not taken seriously.

<u>Cerenkov radiation</u>

It is well known that light travels at different speeds through different media. It moves more slowly through glass or water than it does through air and this is what gives rise to refraction. The change in speed is what determines the refractive index of a medium. When a charged particle moves through a medium with velocity *v* that is greater than the speed of light in that medium, c_m then it emits Cerenkov radiation. Cerenkov photons emerge at an angle θ to the direction of the particle.



$$\cos\theta = \frac{c_m t}{vt} = \frac{c_m}{v}$$

The effect is similar to a sonic boom when an aiercraft goes through the sound barrier.

Cerenkov counters are used to identify high energy charged particles inside reactors and accelerators as they emit this light.

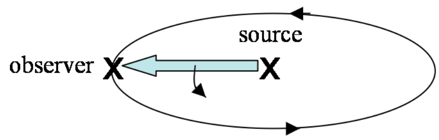
c_m depends on the refractive index. Thus for water, c_m = 0.75c (where c is the speed of light in the vacuum = 3x10^8 m/s).

<u>The searchlight paradox</u>

If a searchlight is rotatingvery fast then the further away from it that an observer is, the faster the beam will sweep past (in a larger and larger circumference).

If the source is aa distance, r, away from you and the beam sweeps round at n revs/sec, how far away do you have top be for it to sweep past you faster than c?

The 'sweeping' covers a distance of $2\pi r$ in $1/n$ seconds.



Therefore, $v = 2\pi rn = 3\times10^8$ m/s

For instance, if n =100 Hz then

$2\pi r \times 100 = 3\times10^8 \quad \Rightarrow \quad r = 500$ km.

This doesn't seem so outrageously large. But there are real examples much more dramatic: pulsars.

Consider the pulsar (rotating neutron star) in the Crab Nebula. It is roughly 7000 LY away from Earth and spins at 30 revs/sec.

$$\therefore v = 2\pi \times 7000 \times \underbrace{3\times10^8 \times 3600 \times 24 \times 365}_{\text{Distance light travels in one year}} \times 30 = 10^{14} c \quad \text{!!!}$$

This seems incredible! But the answer is simple: the beam is made up of indivudual photons arriving at Earth having moved radially outwards from the pulsar and having travelled at a speed c. Nothing physical is actually moving round in a circle. So nothing is breaking the light speed barrier. Two point along the circumference might get photons arriving within a time interval that is so short that no signal could pass between them, say the earlier observer letting the later one know that he has received his photon. The two events are joined by a 'spacelike' worldline and so are not causally connected.