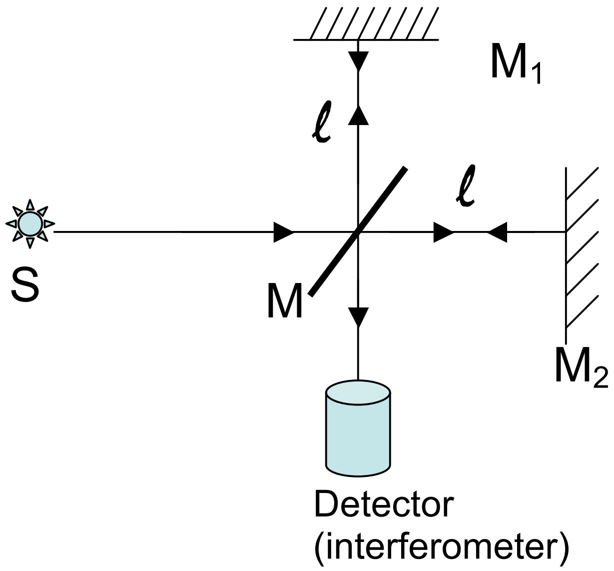
Part II: Special Relativity

The Michelson-Morley experiment

After Maxwell's work on electromagnetism in the 19th century, physicists wanted to understand how em radiation (light) propagated in vacuum. It was suggested that there had to be some medium that pervaded the whole Universe, dubbed the 'ether'.

Two American physicists, Michelson and Morley conducted one of the most famous experiments in the history of science (1887) to prove the existence of the aether. If it exists, they reasoned, then light emitted from a source in the laboratory, which is on Earth and therefore moving through the aether, would be measured to be travelling at different speeds depending on its direction.



Set-up

A light source emit monochromatic light in the direction in which the Earth is spinning (moving through the aether) onto a beam splitter (half-silvered mirror), *M*, placed at 45 degrees to the beam direction. The reflected light bounces off towards another mirror, M_1, a distance ℓ away and is then reflected back down to a detector. The part of the beam transmitted through *M* continues to travel in the original direction to a third mirror, M_2, also ℓ away, bounces back and has a component that is reflected off *M* down to the detected. Both the beams reflected back from M_1 and M_2 have components that head back towards the source, which are not of interest. What is important is the combined beam into the detector. Since, the light in both directions to M_1 and M_2 and back will have travelled a distance 2ℓ, they should reach the detector in phase. This is an interferometer device (now called a Michelson interferometer) that shows fringes of light (bright if in phase, dark if out of phase).

The issue is what distance the light really has travelled. We have

$$MM_1 = MM_2 = l$$

So path lengths are each $2l$. But what if whole apparatus is moving at a speed v in the MM_2 direction due to the Earth moving through the aether?

We would have

Speed of light in MM_2 direction $= c - v$

Speed of light in M_2M direction $= c + v$

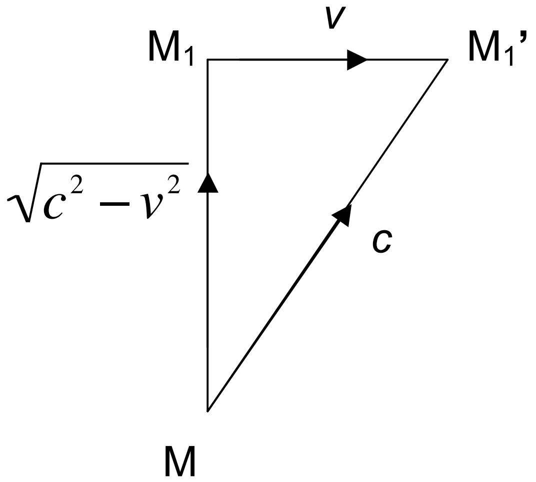
$$\therefore t_2(\text{time for } M \to M_2 \to M) = \frac{l}{c+v} + \frac{l}{c-v} = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1}$$

For light travelling $M \to M_1 \to M$, we must add velocities vectorially.

In the figure, the path MM_1 is the one the light beam is seen to take as viewed in the lab. The path MM_1' is the one seen by an external observer stationary in the ether. Thus,

$$t_1 = 2 \times \frac{l}{\sqrt{c^2 - v^2}} = 2l \frac{1}{\sqrt{c^2}} \Big/ \sqrt{\frac{c^2 - v^2}{c^2}}$$

$$= \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$



For $v << c$ we can use the Binomial approximation for each of the expressions for t_1 and t_2:

$$\left(1 - \frac{v^2}{c^2}\right)^{-1} \approx 1 + \frac{v^2}{c^2}, \qquad \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx 1 + \frac{v^2}{2c^2}.$$

Thus the time difference is

$$\Delta t = t_2 - t_1 = \frac{2l}{c}\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{v^2}{2c^2}\right)\right] = \frac{2l}{c}\left[\frac{v^2}{2c^2}\right] = \frac{lv^2}{c^3}.$$

Now, experimentally they found that $\underline{\Delta t = 0}$.

It followed therefore that $\underline{v = 0}$. That is, there is now velocity of the Earth relative to the aether. Many scientists at the time could not believe this 'null' result. They tried to explain this result while still retaining the notion of the aether since they could not contemplate its non-existence.

Around the turn of the century two physicists, Poincaré andf Lorentz, tried to explain the Michelson-Morley result and almost 'got there'. But in 1905, Einstein reached the correct answer.

He presented two postulates:

Einstein's Postulates

-1-

The laws of physics are the same for all observers no matter how fast they are moving relative to each other.

(e.g. if you are in a rocket with the engines off you cannot say whether you are moving stationary.)

-2-

The speed of light has the same value (*c*) measured by all observers no matter how fast they are moving relative to each other.

(e.g. the speed of light coming from the sun is the same whether measured on Earth or on a rocket speeding away from the Solar System at half the speed of light!)

Poincaré had also reached these postulates in 1904 but unfortunately for him he did not summarise all the basic concepts into one formal theory. That is why Einstein gets the credit. Lorentz and Fitzgerald had proposed a set of transformation equations that predicted a length contraction in the direction of motion (MM_2 in the Michelson-Morley exp) which would explain their null result but allow the aether concept to be saved. They were very close; the Lorentz Transformation equations form the basis of Special Relativity and Lorentz contraction is real but they proposed it for the wrong reason in order to preserve the aether.

Technically, M&M experiment showed not that the speed of light is constant in all inertial frames but rather that it was constant in all directions within the same frame. It was not until 1932 when another famous experiment was carried out (by Kennedy and Thorndike) that Einstein's second postulate was properly proven: that speed of light is constant in *all* inertial reference frames.

In the 1960's experiments at CERN showed this very clearly when studying the decay of the pion. Pions travelling at 99.75% of the speed of light emit γ-rays in the direction of motion that were measured to still have a velocity *c* in the lab. So however fast the pions are travelling, the emitted photons do not come out any faster. The pions see the photons coming out at a speed *c* and so do we.

Einstein's second postulate states that the speed of light is the same in all inertial reference frames (or: speed of light has the same value as measured by all observers no matter what their relative velocity with respect to each other is).

An important consequence of this is the nature of time itself as an absolute quantity is no longer true. This sounds a rather outrageous claim and requires some explanation. So here goes:

When Einstein was 16, he considered what would happen if he were to chase a light beam. He asked himself whether, while flying at the speed of light and holding a mirror in front of his face, he would see his reflection. Since to do so required the light to bounce off his face, into the mirror and then reflect back into his eyes, how could the light overtake his face to reach the mirror if he was travelling at light speed? This is what he answered with his second postulate: he would indeed see his reflection as normal. But has this to do with the nature of time?

Imagine someone (call them A) travelling in a rocket at half the speed of light alongside a beam of light. To an observer (B) on Earth, they would see the light heading off at speed *c*, B moving at *c/2* and so the light overtaking B at *c/2.* But according to postulate 2, A will still see the light overtaking him at a speed *c.* Even if A reached 99.9% the speed of light (we will see later on how nothing can travel AT light speed apart from light itself) then B would see the light beam just creeping past A as it is moving ever so slightly faster. However, A still sees it overtake him at a speed *c.* [Note that in all this I have taken $c=3x10^8$ m/s.]

How can this be? Who is right? The answer is that both are right in their own reference frames. But something has to give. In this case it is the nature of time itself. B will see A's clocks running slower than his. One second on board the rocket lasts a long time during which the light beam can cover a significantly greater distance than it does in one Earth second. So B will see how, in A's slow motion time, the light beam would appear to still be travelling at *c*.

So, the second postulate of Einstein's manifests itself in two ways involving the nature of time:

<u>Relativity of simultaneity</u>

Simultaneous events according to one observer may not be simultaneous to another (the relativity of simultaneity). An important point to make here is that this is not just a consequence of the finiteness of the speed of light (i.e taking different durations to reach two observers. That is obvious and can easily be accounted for without any notions of Einsteinian relativity. Rather it is a consequence of the second postulate about the constancy of the speed of light.

You may wish to refer to the diagrams on the following pages while reading through this discussion.

A and B are on board rockets moving towards each other at constant relative velocity. Two events (flashes of light) take place at either ends of A's rocket: E_F (event at front) and E_R (event at rear). Will A and B agree on the ordering of these two events?

Begin first by considering B's reference frame (the one in which B's rocket is at rest – or the one moving alongside B's rocket). Let us say that at time T_0 B and A are alongside each other. At some later time T_2, B sees the light from the two events, E_F and E_R, reach him simultaneously. So he calculates that since light travels at constant speed from both events, they must both have happened at the same moment, T_0. B will argue that the position of the events in his frame do not change

The relativity of simultaneity I

Note that length contraction has not been taken into account in the drawing. According to B, A's rocket should be shorter than it is due to length contraction.

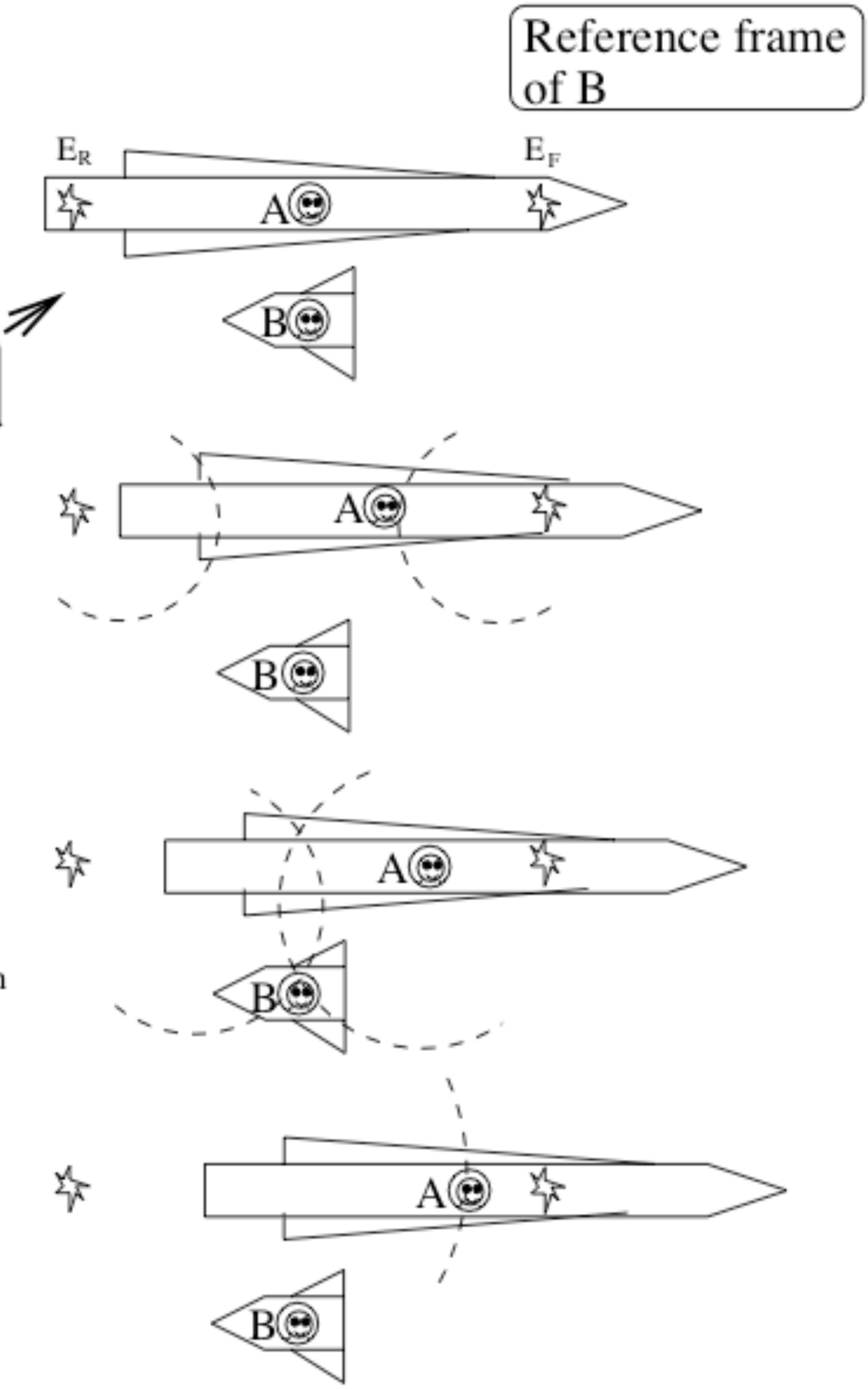
At T_0 neither A nor B is aware anything has happened yet.

E_R is event at rear of A
E_F is event at front of A

At T_1 B says A should see light due to event E_F.

At T_2 B sees light from both events E_F and E_R arriving simultaneously. So events must have been simultaneous.

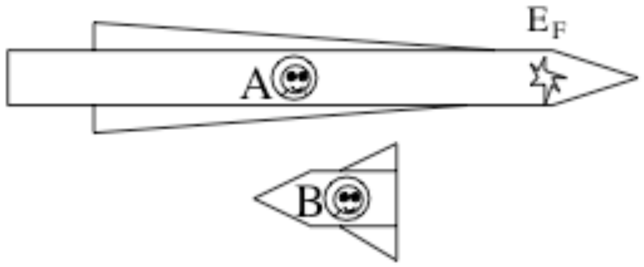
Finally, at T_3, light from E_R should reach A.



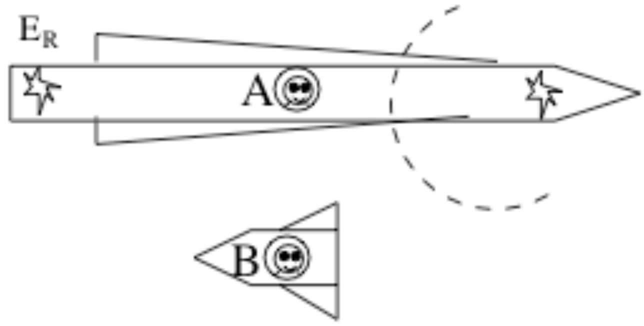
The relativity of simultaneity II

Reference frame of A

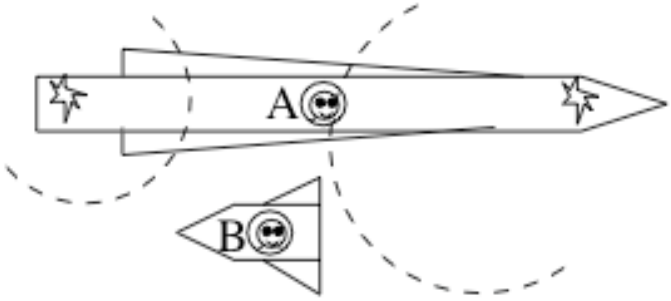
Event E_F at T'_0



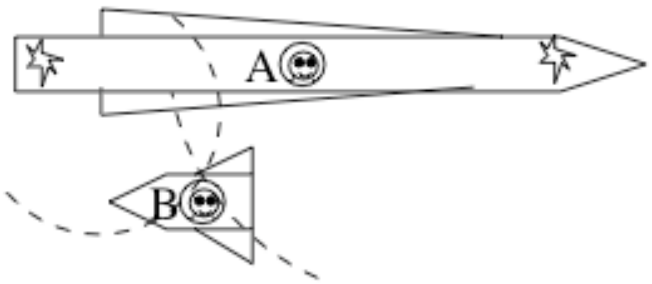
Event E_R at T'_1
Neither A nor B have seen anything yet.



At T'_2 A sees light from E_F. But since B is moving away from it he has yet to see it.



B is now much closer to E_R so although it occurs at a later time it reaches him at same time as light from E_F. So A only sees events as simultaneous because he is moving.



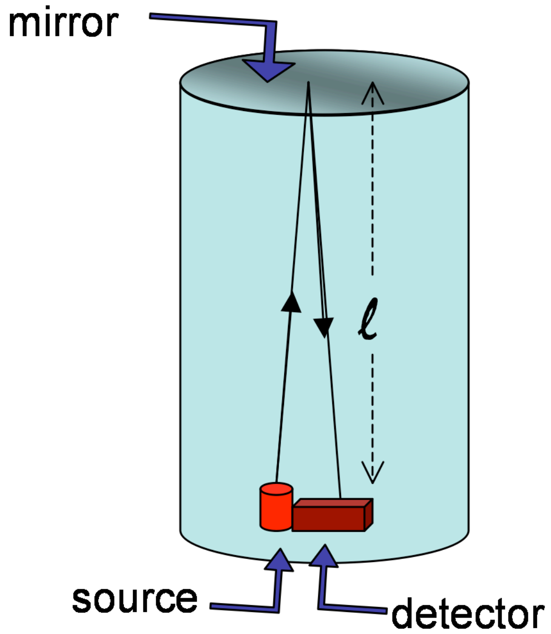
but A's rocket was positioned at the right place at T_0 when B was half way between the front and back of A's rocket. Of course A's rocket moves on but the location of the events in B's frame are fixed. B therefore predicts that A will move towards the place at the front of his rocket where E_F happened and so will see it before he sees E_R, which he is moving away from. So B sees them together but understands why A doesn't.

In A's frame, he claims he is always half way between the front and back of his rocket and so light will take the same duration to reach his eyes. Of course he confirms B's statement that he does not see the events simultaneously but this is because E_F really does happen before E_R. He argues that the only reason B sees these two sequential events at the same time is because it is B who is moving towards E_R and so the light from that event doesn't have so far to travel. The light from E_F, which set out earlier is chasing B and happens to reach his eyes at the same time as E_R., they both agree that one of them sees events simultaneously and the other doesn't. They both have sensible explanations for why. But their explanations are different. The issue is, never mind what A and B think, *did the events happen simultaneously or not??*

The constancy of the speed of light tells us therefore that there is no right answer. The simultaneity of events simply depends on ones reference frame.

<u>The slowing down of time</u>

We can measure time using a 'light pulse clock'.

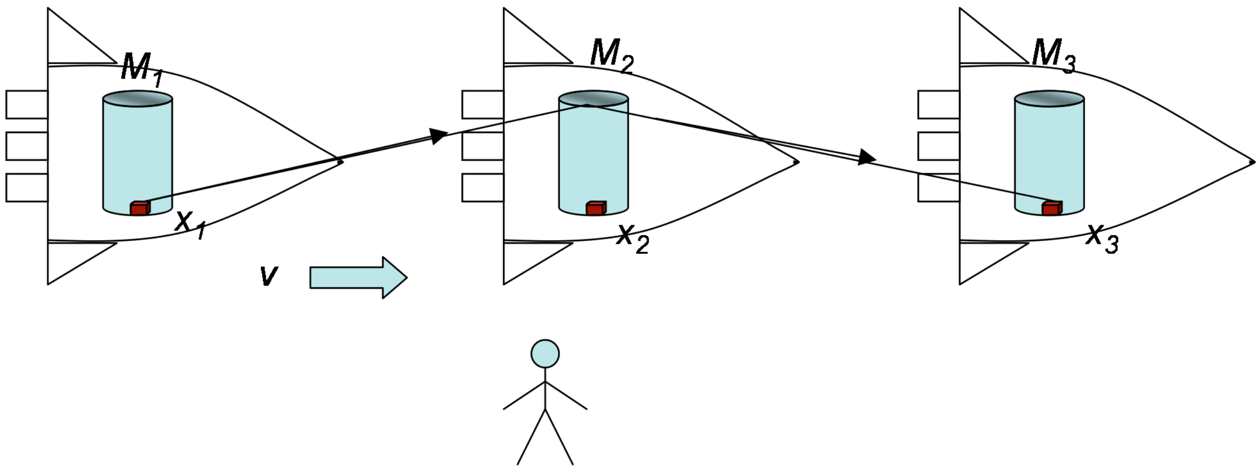


A pulse of light is emitted by a source at the bottom of the clock towards a mirror at the top and is reflected back down to detector.

We assume that to a good approximation, the light pulse goes straight 'up and down' so that in the rest frame of the clock it covers a distance of 2ℓ and so time taken to go there and back is

$$t_c = \frac{2l}{c}$$

We now put the clock on board a rocket and observe it zoom past. We will discover that by simply applying the notion of the constancy of the sol we can prove that time does indeed have to be running at different rates. Imagine an observer watching the rocket with his own clock that can accurately measure the time it takes for the pulse on the rocket clock to leave the source and return to the detector. Call this time t_0.



Position of source/detector at start and finish are x_1 and x_3.

$$\therefore x_1x_3 = vt_0 \quad , \quad x_1x_2 = \tfrac{1}{2}vt_0$$

$$x_1M_2 = \sqrt{l^2 + \left(\tfrac{1}{2}vt_0\right)^2}$$

Therefore the distance travelled by the light pulse according to the stationary oberver is $2\times\sqrt{l^2 + \left(\tfrac{1}{2}vt_0\right)^2}$.

But in the observers frame, the light is still travelling at a speed *c.* However, now it has to cover a longer distance. It does this in a time

$$t_0 = \frac{2\sqrt{l^2 + \left(\tfrac{1}{2}vt_0\right)^2}}{c}$$

As you see*,* t_0 appears on both sides. Solve for t_0 to give

$$t_0 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

and substituting, $t_c = \dfrac{2l}{c}$

we have

$$\underline{t_0 = t_c\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}}.$$

Since the factor multiplying t_c is always greater than 1 (check this) then we have

$$\underline{t_0 > t_c}$$

So observer will see that more time has elapsed on his clock (t_0) than has elapsed on board the rocket clock (t_c).That is, he sees the rocket clock running slower.