Part III: General Relativity

The Principle of Equivalence

In 1907, Einstein had the 'happiest thought' of his life:

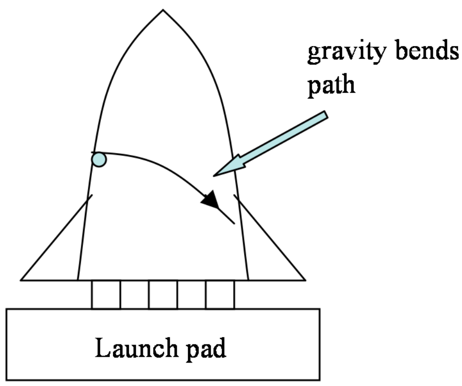
For an observer falling freely there exists in his immediate surroundings no gravitational field. If he releases an object while falling then it will float alongside him as they both drop together (that is, it will be at rest in his accelerating – non-inertial – reference frame). In addition, he will feel the same sensation as he would if floating weightless out in space. Einstein realised that while accelerating under the influence of gravity the effects of the force of gravity disappear: the acceleration due to gravity cancels out the 'pull' of gravity. In the same way, we can put back the effects of gravity via acceleration.

That is, the effects of gravity and acceleration are equivalent. This is known as the PRINCIPLE OF EQUIVALENCE.

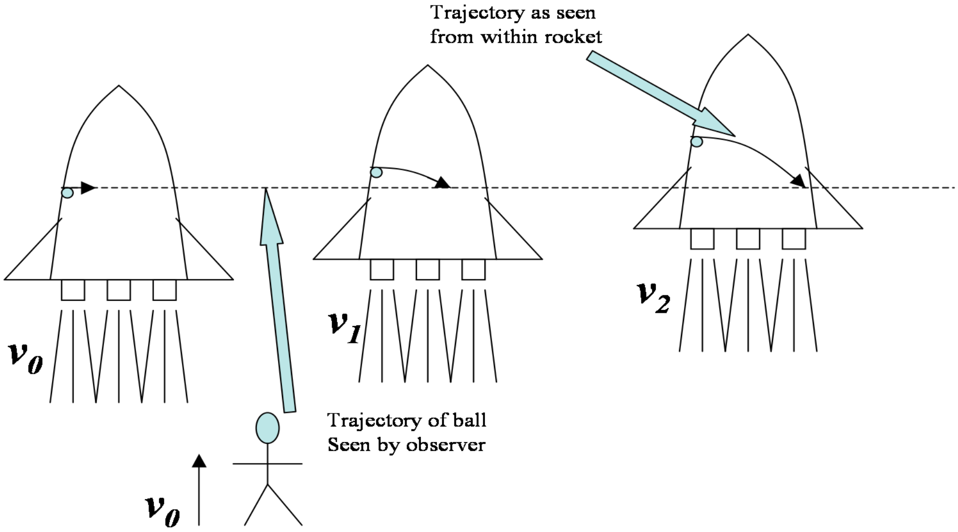
Imagine a rocket with no windows floating in space. SR says that we cannot tell if it is standing still or moving with constant velocity, since it depends on what inertial reference frame one is in. Einstein's P. of E. states that if you feel a force pushing you down onto the floor of the rocket equal to mg (where m is your mass), then you cannot tell if you are accelerating upwards at $1g$ out in space or stationary on the surface of the Earth. An example of where this can be demonstrated is in theme park simulator rides, where the effects of acceleration are simulated by gravity.

We can demonstrate this equivalence more carefully by conducting an experiment inside a rocket:

If a ball is thrown across the rocket horizontally while it is still stationary on Earth, then the Earth's gravity will give it a curved trajectory downwards.



On the other hand, if the rocket is accelerating out in space at 1 g then we need to investigate more carefully how the trajectory looks. We must do this from two reference frames. Imagine an observer floating alongside the rocket while it is travelling at constant velocity $\underline{v}$.



The outside observer travelling at initial velocity of rocket sees ball released horizontally and therefore with no vertical component to its velocity. At that instant, the rocket accelerates at 1 *g* and so the three diagrams are successive time frames showing the rocket edging away from the observer. But since the ball, while inside the rocket, is nevertheless not involved in this acceleration, it will continue to move horizontally according to the outside observer, but the back of the rocket moves up towards this horizontal line of travel of the ball.

Inside the rocket on the other hand, it would appear as though the ball is moving in a downwards trajectory, tracing a path exactly as it would when under influence of Earth's gravity.

We now do same experiment but by shining a light beam across the rocket. Consider first scenario 2 (the accelerating rocket). For exactly the same reasons as before, the light beam will appear from within the rocket to bend slightly downwards (since the rocket is accelerating upwards towards it). The P. of E. states then that the same bending of the light beam should be seem when the rocket is still on Earth!

The bending of the path of the ball is not surprising in both cases, but the bending of the light beam under influence of gravity probably is. After all, light is massless so how can gravity pull it down?

One answer mighty be to say that since mass and energy are equivalent then the lights energy behaves like an effective mass that can be inlfuenced by gravity. But if one were to do the sums then the amount of bending calculated would be out by a factor of two.

There is a better and deeper explanation. Gravity bends spacetime itself. Balls, light beams and everything else are then just tracing curves through spacetime. In fact, they trace straight lines as best as they can that become curved when the medium they are travelling through (spacetime) is bent. Such paths are known as <u>geodesics</u>. They are defined as the shortest distance between two points in a curved space.

All bodies curve the spacetime around them. One of the most famous tests of GR (and the one that made Einstein famous) measured this curvature and its effect on a light beam due to the gravitational field of the Sun (see later).

There are two ways of defined what General Relativity is about:

1. It generalises SR to include accelerating frames (not just inertial ones). So SR is just a special case of GR.
2. It is about the geometry of spacetime and how it is affected by gravity (i.e. a deeper origin to gravity than what Newton tought us).

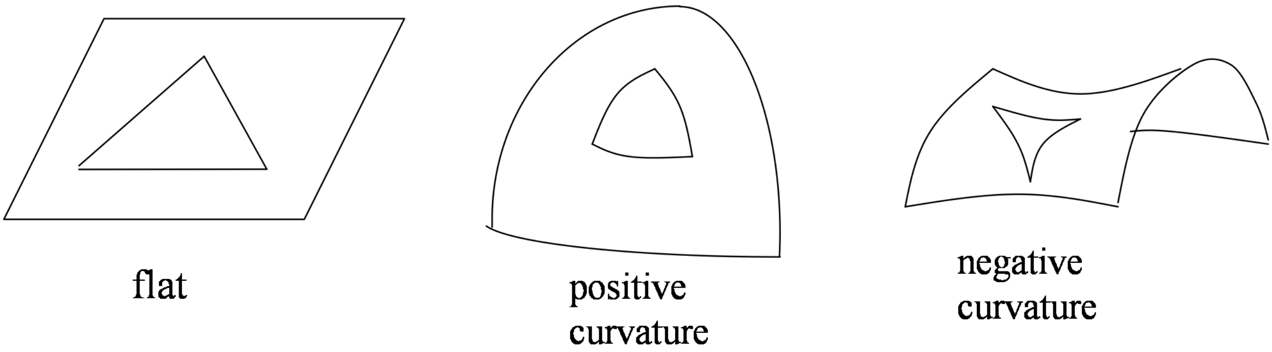
Curvature of spacetime

To understand this, we need to simplify the picture and look at 2D space. When flat, the rules of geometry are called Euclidean. But if surfaces are curved then the rules of Eulidean geometry break down.

For instance, the sum of interior angles of trangles $\neq 180^\circ$

And the circumference of circles $\neq 2\pi r$.

There are two types of curvature: <u>positive</u> and <u>negative</u>.



In a positively curved space (surface), interior angles of a triangle > 180 degrees

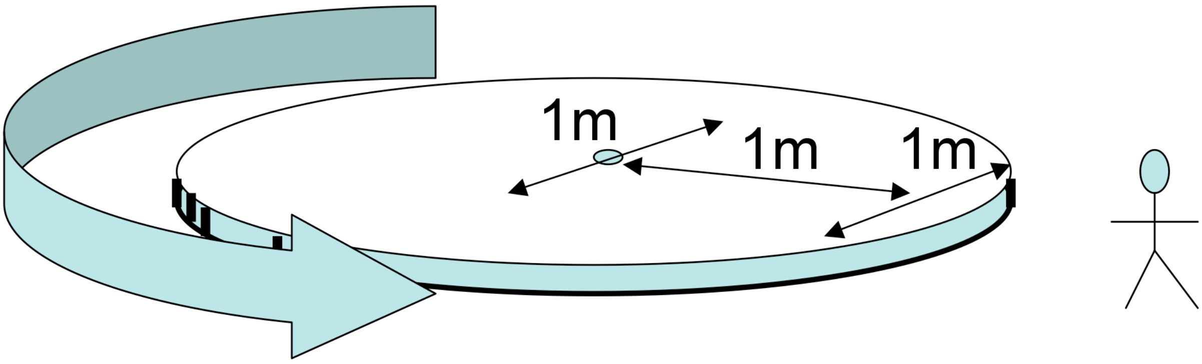
In a negatively curved space (surface), interior angles of a triangle < 180 degrees.

Locally, in region of concentration of mass/energy, spacetime is curved positively. But for Universe as a whole, it might be flat or even negatively curved. We simply do not know yet what the overall shape of the Universe is yet.

Not only does mass and energy cause spacetime to curve but if it oscillates/vibrates then it can create ripples that propagate through spacetime at the speed of lifght. These are known as gravity waves and, while not yet confirmed experimentally, are believed to exist and are predicted by GR.

<u>The spinning disk</u>

Put three metre rods on a spinning disk of roughly 1m radius in the positions shown in the diagram below. If the disk is spinning close to light speed then to an outside observer (off the disk) the rod on the disk's perimeter will appear length contracted as it moves tangentially past him.

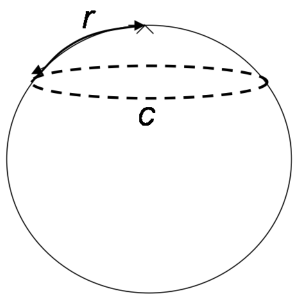


For another observer standing at the very centre of the disk, there is no acceleration (ignore the 'spinning on the spot') and so is in an inertial frame. Since he is stationary to the outside observer (while spinning, he is nevertheless not moving from his location) and is therefore in the same inertial frame as the outside observer. Thus he should agree with him that the rod on the perimeter is shorter. However, he can be facing this rod at all times and it therefore willl stay stationary relative to him (one radius away at all times). So he cannot use special relativitistic length contraction to explain its shortening.

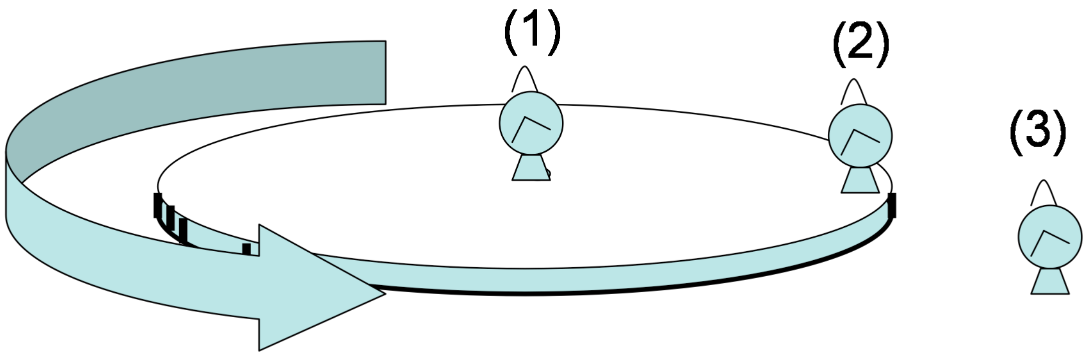
Instead, if he were to walk towards the rim of the disk, he will find it increasingly hard not to be flung off (ignore the fact that it is spinning near lightspeed and he would in practice have been flung off straight away). This is because acceleration increases as he moves radially outward. And since acceleration is equivalent to gravity then we see that it is through the Principle of Equivalence that gravity also seems to have an effect on lengths. This is however because of the way it bends space. Thus the person at the centre of the disk sees the perimeter still 1m away but having a circumference that is less than $2\pi r$.

This can best be understood if we go back to look at a curved 2D surface. The view of the observer at centre of disk is similar to that of an observer at the North Pole measuring the circumference of the Arctic Circle.

The radial distance form the pole to the circle would, if flattened out, give a circle of circumference $2\pi r$, which would be larger than the dashed circle draw. Thus this Arctic Circle traces a distance that is less than $2\pi r$.



We can also see the effect of gravity on clocks by using the P. of E. in the same way as before and considering the spinning disk.



Clocks (1) and (3) run at the same rate (not moving relative to each other so no SR time dilation and clock (1) is not accelerating as at centre of disk, so both in same inertial frame. Clock (2), however, will run slower than (3) as seen by observer at clock (3) due to time dilation since it is moving. Thus, observer at centre should see (2) running slower than (1). This is because clock (2) is accelerating. And since gravity and acceleration are equivalent then gravity should also slow time down.

Of course we cannot untangle the effects of gravity on space and time and must deal consistently with its effects on 4D spacetime. But it is often easier to see how it curves space and slows down time separately; we just have to be careful to remember that they are not independent effects.

Experimental Tests fo General Relativity

1. <u>Advance of Mercury's perihelion</u>

The planet Mercury follows an elliptical path around the Sun. Other planets, however, pull at it and perturb its motion so that its axes slowly rotate in its plane.

The point along its orbit when it is closest to the Sun is known as the 'perihelion'. The shifting of this point around gradually as it follows many orbits is called the procession of the perihelion. Calculations using Newtionian mechanics predict a procession of 532 seconds of arc per century. (This means it takes about a quarter of a million years for the procession to complete a full orbit.) However, experimentally this procession has been observered by astronomers in the 19th century to be 43 seconds of arc more than ther calculated value. Such was the scientists' confidence in these numbers that they proposed the existence of an undetected planet dubbed 'Vulcan'(!) on the other side of the Sun to Earth whose gravity added the extra perturbation necessary to account for the discrepancy.

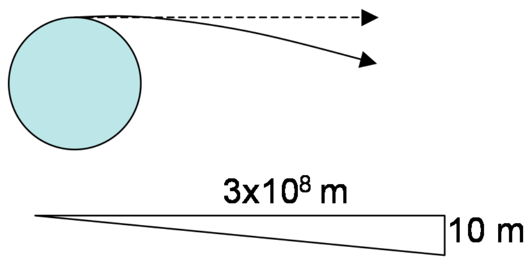
One of the first things Einstein did with his new general theory of relativity in 1915 was to recalculate this effect by considering Mercury's path as a geodesic in spacetime. The value he obtained matched the experimental one perfectly.

Of course it should be stated that measuring the experimental value was a great feat in itself since the Earth also has a small residual perihelion advance, in addition to its wobbling motion due to the effects of the Moon.

2. Bending of a light beam

Can we detect the bending of light by Earth's gravity?

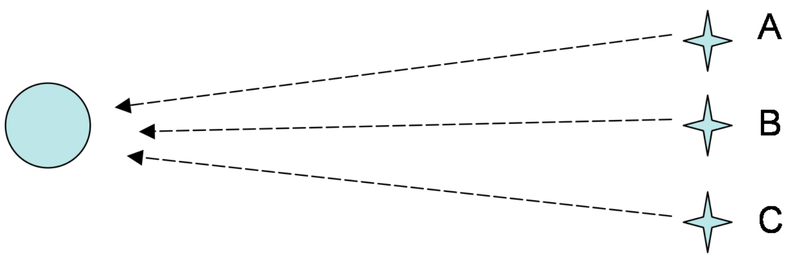
Shine a torch out into space. In one second light travels a distance of $3x10^8$m horizontally and 'falls' 10 m due to Earth's; gravity (*g).*



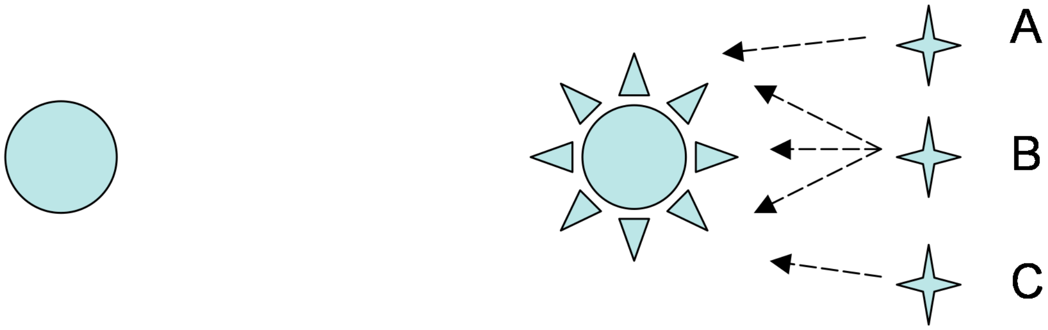
Therefore effect is too small to detect if we wished to carry out experiment on Earth.

But the Sun's gravity *is* large enough.
In 1919, a famous expedition set out for the Equator to observe a total eclipse of the Sun and test the predictions of GR.

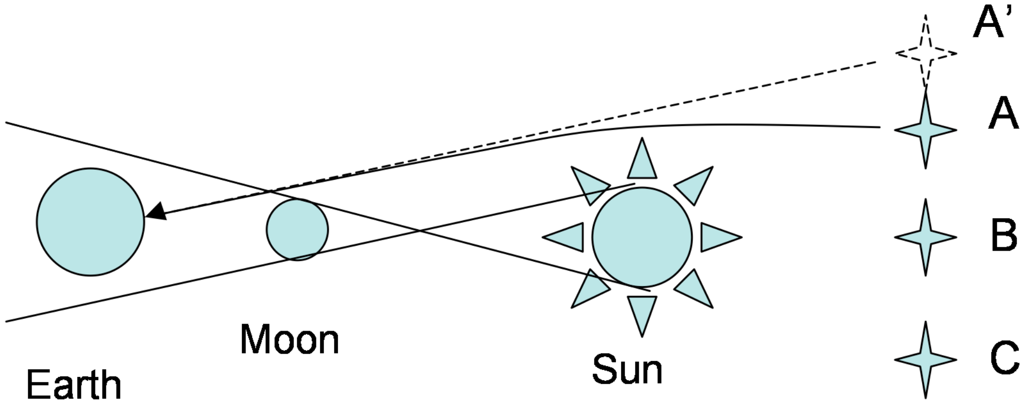
Consider three stars shine next to each other (It doesn't matter how far away they are just that they appear close together in the night sky.



However, if the Sun is in the way its gravity should affect the path of light from the stars. But saying that the Sun is in the way means that it would be daylight and the stars are obscured.



During a solar eclipse, however, the Sun's rays are blocked by the Moon



Light from star A, which should not be able to treach Earth now gets bent by the Sun's gravity and can be seen of the Sun's light is blocked. However, on Earth we now see star A in a new 'apparent' position A'. This shift in its position depending on whether the Sun's gravity is bend its light or not can be translated in curvature of spacetime around the Sun.

The results of the experiment confirmed Einstein's prediction spectacularly and he instantl;y became a household name around the world.

3. Gravitational redshift

GR also predicts that time runs slower in a gravitational field (as it is spacetime that is bent, not just space). The stronger the field, the slower time runs. Einstein calculated that clocks tick more slowly on the surface of the Sun than they do on Earth. If we could put two identical clocks on Sun and Earth then we would see that for each second that elapses on the Sun, 1.000002 seconds go by on Earth.

Of course, we can't put clocks on the Sun but we have natural clocks at our disposal: gamma rays emitted by nuclei in the Sun. When atomic nuclei are excited they will de-excite by emitting a gamma ray of a definite energy (since their energies are quantised). From Planck's relation we know that this energy is proportional to frequency of the gamma ray. Since $f=1/T$ then frequency can be regarded as a measure of time. If a nucleus that gives out a gamma ray of specific frequency is placed in a gravitational field, the slowing down of time wil cause the frequency to drop. So we would expect gamma rays from the Sun (indeed all em radiation) to be redshifted. This is known as the <u>gravitational redshift</u> and is not the same as Doppler shift.

American physicists, Pound and Rebka measured this effect on Earth and proved that time does indeed slow down, even on Earth (1959, 1965). They measured the redshift of gamma rays at the top of a 74 foot tower that had been emitted by cobalt nuclei at the bottom of the tower.

Their experiment was possible thanks to the then newly discovered Mössbauer effect: identical nuclei emit and absorb light of the same frequency. But if this frequency is changed even slightly then the light can no longer be absorbed by nuclei identical to the emitters. Pound and Rebka placed cobalt nuclei at the top of the tower and found that they no longer absorbed the gamma rays emitted from the bottom due to a slight gravitational redshift arising from the slightly stronger gravitational field. They then employed a very clever trick: by moving the nuclei at the top of the tower downwards at a controlled velocity they enduced a Doppler (blue shift) in the gamma rays (since in the rest frame of the falling nuclei, the source of the gamma rays – the nuclei at the bottom – is seen to be moving towards them). Their falling velocity and hence the blueshift could be adjusted to exactly cancel out the gravitational redshift and bring the gamma rays back to the correct frequency to be absorbed. They calculated from this method the amount of gravitational redshift and this agreed with the predictions of GR to within 1%.

Black Holes

In such a brief introductory course on General Relativity it is impossible to go into any depth. So having covered the Principle of Equivalence, the curvature of spacetime and the experimental tests of GR, we skip any discussion of Einstein's field equations (the equations of GR that link mass/energy to curvature of spacetime) and end with a look at an extreme and exotic (and yet experimentally – almost – confirmed) consequence of GR: the black hole.

How do black holes form? To answer this we need to study stellar and galactic evolution. Stellar black holes form when a massive star, of several solar masses, collapses under its own weight once it stops shining at the end of its life. Basically, its falls down its own gravitational field! Supermassive black holes form at the centres of large galaxies very early on in their evolution (quasars) and have a mass typically of millions of solar masses.

A black hole is a region of spacetime when matter is so densely packed that its gravity has warped spacetime to an extent that not even light can escape from its pull. At the centre of a black hole is the singularity; in the simplest picture this is a point of zero size and infinite density (although quantum mechanics saves us from this limit by insisting on a tiny amount of uncertainty or fuzziness).

Surrounding the singularity is the event horizon. This is the surface of an imaginary sphere of a certain radius called the Schwarzchild radius. The horizon marks the point of no return as you approach a black hole; once you get closer to a black hole singularity than the Schwarzchild radius then there is no escape. Since this is also the region from which light cannot escape, it also defines the size of the black hole (that is the extent of the 'blackness') as seen from outside.

There is a very simple way of deriving a formula for the Schwarzchild radius from simple Newtonian ideas.

We begin by considering the notion of escape velocities. For a body of mass m to escape the pull of a massive object such as a star or planet, it must acquire a kinetic energy in excess of the gravitational potential (binding) energy of the object. The escape velocity, v_e, is the critical velocity for this. Thus,

$$\tfrac{1}{2}mv_e^{\,2} = \frac{GMm}{r}$$
$$\Rightarrow v_e^{\,2} = \frac{2GM}{r}$$

Clearly, as mass M increases, so does the necessary escape velocity. But more interestingly, if we keep the mass constant we see that reducing the radius also increases the escape velocity. Once the radius falls below the Schwarzchild radius r_0 the escape velocity exceeds the speed of light.

At the critical radius, $r = r_0$ we have $v = c$. Therefore

$$c^2 = \frac{2GM}{r_0}$$

Therefore,

$$\boxed{r_0 = \frac{2GM}{c^2}}$$

We see in this simple and elegant formula that we have two univeral constants: *G, c*. Thus, the size of a black hole is related purely to its mass and its radius is in fact proportional to the mass: double the mass, double the radius. Or, if two black holes of equal mass M collide, the will form a single balck hole with twice their radius.

Example:

What if the Earth could be compressed down to form a black hole? How large would it be?

We simply substitute in numbers to the formula above:

$G = 6.7\times10^{-11}$ m^3 kg^{-1} s^{-2}, $c = 3\times10^{8}$ m s^{-1}, $M = 6\times10^{24}$ kg =>

$$r_0 = \frac{2\times6.7\times10^{-11}\times6\times10^{24}}{\left(3\times10^{8}\right)^2} = 9\text{ mm.}$$

What is the physics of a black hole? How do objects behave near the horizon?

This is a very difficult subject and certainly, once inside the event horizon, we cannot be sure as the laws of physics breakdown. Things get very weird. For instance, spacetime is so twisted that the radial direction of in-falling objects is no longer a direction in space but is instead the direction of the time axis! This is why all objects must always fall radially inwards: any movement away from the singularity constitutes motion backwards in time. It is just as inevitable that you will hit the singularity if you fall into a black hole as it is that you will reach tomorrow. Also, we see now that the Schwarzchild radius, which is in units of spatial length is only meaningful as a measure of size of the black hole *from outside*. This is not the true size of a black hole and things are very different inside.

The singularity of as black hole marks the edge (end) of spacetime. There is no space or time beyond it. This is the opposite to objects called white holes, whose singularity marks the beginning of spacetime. Our universe was born from such a singularity at the Big Bang.

Consider now two observers: one who suicidally jumps into a black hole while the other watches from a safe distance outside the horizon. Firstly, consider the view of the falling in observer:

1. He will not know exactly when he has passed the event horizon (only if he tries to turn back).
2. If he looks back the way he came he will see events in the outside universe taking place in speeded up motion since his time is running slower (although he is unaware of it)
3. The light from the outside universe will be focussed increasingly into a ring and it will be as though he has entered a tunnel with the entrance receding and getting smaller. This is because light from all the stars around the black hole gets bent round and focussed.
4. Depending on the size of the black hole, the journey from the horizon to the singularity will take, according to the clock carried by the falling observer, from seconds (stellar black hole) to hours (supermassive black hole).

From the perspective of the outside observer:

1. The falling observer will appear to fall more and more slowly towards the event horizon as the gravitational field increases in strength and slows time down. At the horizon there will be maximum gravitational time dilation and the falling observer will appear to be frozen, floating just at the horizon [whereas as, as far as the falling observer is concerned, he has actually been falling faster and faster and accelerates through the event horizon at the speed of light!]
2. Light from the falling observer that reaches the outside observer will be more and more redshifted until it quickly falls to frequencies beyond the visible. Thus in practice the faller will never be seen frozen at the horizon but will have gradually faded away by then.
3. This redshift of the light can be thought of in another way. Not only will the wavelengths become too long for us to ‘see’ them but the stretching of the wavelength comes from the stretching of spacetime itself. Thus the photons have further and further to travel as the fall closer to the horizon in order to escape the gravitational well. The final photons to leave the body of the falling observer before he disappears beyond the horizon will be infinitely redshifted and take an eternity to reach the outside observer.