UNIVERSITY OF SURREY

Department of Physics

Level 1 Module

Space, Time and Relativity

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Part I: Space and time before Einstein

Introduction

In 1905 Einstein showed how Newtonian mechanics broke down when applied to very high-speed motion. Let us first briefly examine Newtonian mechanics and its foundations to see what could be wrong with it.

Define "event":

A physical event is something that happens independently of the reference frame we might use to describe it (e.g. collision of two particles, turning on a light source etc.) It happens at a point in 3-D space and at an instant in time. It therefore requires four numbers to define it (*x,y,z,t*). But the important thing to remember is that these four numbers are not universal. They depend on our frame of reference. We have to define a reference frame first and then the event is fixed in *that* frame.

For example, the collision of two particles might happen, according to an observer in the lab at $x=1$m, $y=4$ m, $z=11$ m and $t=7$ secs. (i.e. 7 seconds after some predefined time axis origin). But according to another observer, these numbers can be different: he might also be in the lab but defines the origin of his reference frame in space somewhere else, or his origin in time might be a different event to the first observer. He might be outside the lab and moving and so his reference frame might have at one time shared the same origin as the one in the lab but since he is moving, his coordinates will be different at the time of the collision.

Inertial frame:

This is a frame of reference in which Newton's first law holds, i.e. a body that is *not* acted upon by an external force will be seen, in such a system, to be either standing still or moving at a constant velocity.

If a body is acted upon by an external force it will accelerate. So any reference frame in which such a body still looks like it is not moving or moving at a constant velocity must itself be accelerating and is therefore not an inertial frame.

Any other reference frame that is moving at a constant velocity (remember this means at a constant speed in a straight line) with respect to our inertial frame is itself therefore also an inertial frame.

We can define new inertial frames by simple translations in space and time. The simplest case is a translation in space, so that a body with velocity $\boldsymbol{v}$ in the first frame still has velocity $\boldsymbol{v}$ in the new one. We can also get a new frame by a rotation of the original one by some angle about one or more of the *x,y* or *z* axes. Thirdly, we can also have a new inertial frame moving at a constant velocity relative the original one.

Conclusion

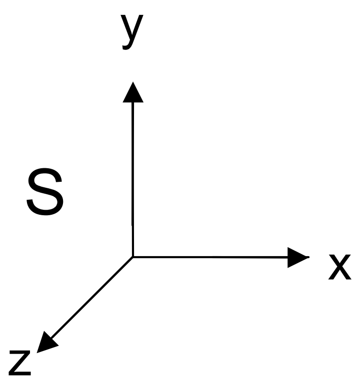
Absolute values of (x,y,z,t) of an event are not important, since we can choose the origin of space and time arbitrarily. What is important is the relative positions in space and time *between different events.*

So, $$\Delta t = t_2 - t_1$$

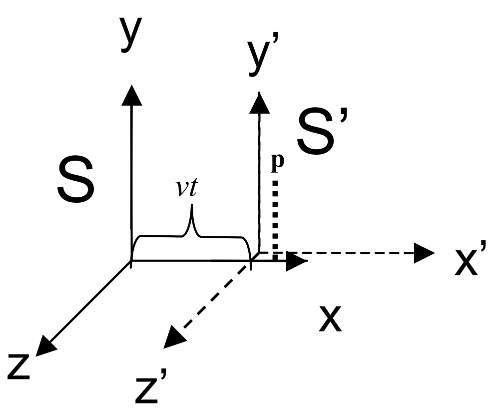
and $L^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$ (i.e. distance between two points in 3-D).

Galilean Transformation equations

We wish to connect the spacetime coordinates of an event in one frame to those in another. Start with inertial frame *S*. It has x-, y- and z-axes. Now consider another inertial frame, *S′*, moving relative to the first. To simplify the problem we choose it to be moving in the x-direction at a velocity **v**.



Consider that the origins of the two frames coincide at time *t=0.* Then at a time *t* seconds later, the frame *S′* will have moved to the right some distance such that its origin is now vt away from the origin of *S.*



Now, any point, p, with coordinates (*x,y,z*) in *S* will have coordinates (*x′,y′,z′*) in *S′* at time *t.* Thus,

Galilean Transformation Equations

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \end{cases}$$

Note also that it goes without saying that $t = t'$.

Space intervals:

A space interval is defined as the distance between two points A,B.
In the frame *S*,

$$\Delta x = x_B - x_A$$

In *S′* then,

$$x'_A = x_A - vt$$

$$x'_B = x_B - vt$$

Therefore, $$\Delta x' = x'_B - x'_A = (x_B - vt) - (x_A - vt) = x_B - x_A = \Delta x$$

So, the distance is the same in both frames, as we might expect.
Of course if *S′* had been rotated relative to *S* such that their x-axes were no longer pointing in the same direction then $\Delta x' \neq \Delta x$. Instead we would need to define the space interval in terms of all three axes.

The space interval will now $= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{\Delta x'^2 + \Delta y'^2 + \Delta z'^2}$ = constant.

<u>Velocity transformation:</u>

If we differentiate the Galilean transformation equations with respect to *t*

$$\frac{dx'}{dt} = \frac{dx}{dt} - v$$

and since *t′* = *t* then $\frac{d}{dt} = \frac{d}{dt'}$ => $\frac{dx'}{dt'} = \frac{dx}{dt} - v$

Therefore,

$$\boxed{\begin{aligned} u'_x &= u_x - v \\ u'_y &= u_y \\ u'_z &= u_z \end{aligned}}$$

These are the <u>*Addition of Velocities Equations*</u>.

<u>Example:</u>

S frame is a the station platform
S′ frame is the moving train

If you walk down the train and the observer on the platform sees you moving at u_x=100 km/hr then, assuming the train is travelling relative to the platform at 95 km/hr, according to the train frame (i.e. to someone seated on the train) you are walking at a speed of 5 km/hr.

What about acceleration?

$$a = \frac{du}{dt}$$

and $$a' = \frac{du'}{dt'} = \frac{du'}{dt} = \frac{d}{dt}(u - v) = a - 0 = a$$

(since v is constant).

So observers in both frames will see same acceleration of a body despite moving relative to each other. This is because while they see different velocities $u' \neq u$, the *change* in velocity seen by both is the same.

Since in classical physics, mass is unaffected by motion (this may seem obvious) then the product ma, is the same for all observers (i.e. in all inertial reference frames). So any force $F=ma$, will have the same value whoever measures it.

Thus Newton's laws of motion and equations of motion are the same in all inertial reference frames. And since the conservation laws (energy, momentum, angular momentum, etc) follow from the laws of motion then all laws of mechanics are the same in all inertial reference frames.

A word of caution here however; since velocities will have different values in different frames, so will momenta and kinetic energies. That is, different observers will get different values for the momentum, say, of a moving body, BUT they will always agree on whether momentum is conserved (in a collision say) or not. If it is conserved they will get different values for the total momentum of a system but at least each observer's values will be the same before and after the collision (the conservation bit).

Newtonian Relativity

The consequence of all this is that we cannot design a mechanical experiment in one frame that would tell us what the motion of that frame is relative to any other. For example, on a plane that has reached cruising altitude and speed, everything will look normal and behave as though the plane is not moving: the way a ball bounces or a pendulum swings, objects fall, collide etc. will all do so in exactly the same way as they would if we were conducting the experiments on the ground. The stewardess does not have to start pouring your cup of coffee at the front of the plane, with you seated at the back in order to take into account the time it takes for the coffee to drop vertically reaching your cup as it and the plane move forward to meet the coffee!

So, no inertial frame is preferred over any other.
=> No physically definable 'absolute' reference frame.
All motion is relative and depends on your frame of reference.
*This is **Newtonian Relativity.***

Transformation laws between frames change some things (coordinates of points, velocities, momenta) but leave others the same (mass, acceleration, force, Newton's laws of motion). Unchanged quantities are called **invariants of the transformation**.

A statement about what the invariant quantities are is called a Principle of Relativity.

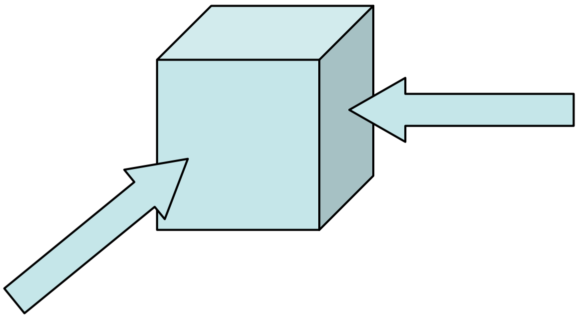
All this seems straightforward enough and correct. But Einstein showed that it was not true. He found that space and time get mixed up and that Δx and Δt are not constant in all reference frames. Instead:

$\Delta x'$ is a function of both Δx and Δt

and $\Delta t'$ is a function of both Δx and Δt.

Only when we combine space and time together into four-dimensional spacetime do we recover a quantity that is constant in all inertial reference frames!

It is as though different observers see lengths and time intervals differently. Think of a box view by you and someone else looking at it from a different angle.



You see the side facing you as square and the side facing the other observer as rectangular (squashed). He will see the opposite. But you would both agree it is just a matter of perspective. We shall see in special relativity (SR) that that it is only in 4D spacetime that we can agree on lengths and time intervals being just 'a matter of perspective' (different points of view from different inertial frames). You and another observer moving relative to you will never agree on the length of an object no matter what angle you look at it, and you will never agree on time intervals between events. Only by uniting space and time do you retrieve an invariant quantity.

So for the moment we have a Principle of Relativity for the Newtonian Laws of Mechanics. But this is not the whole of physics.

<u>Electromagnetism</u>

We have seen how the laws of mechanics are invariant under Galilean transformations. How about the laws of electromagnetism? If they are also invariant then the Principle of relativity will hold for all of physics (at least all of physics known at the time of Einstein). There really would be no experiment that could pick out a preferred inertial reference frame and say 'this is the one that is truly at rest'.

Consider light waves:

As you know, all waves need a medium to travel through (e.g. sound needs air, or other matter, water waves need the water). How about light? Outer space is empty so how does sunlight reach us? Scientists in the nineteenth century argued that there had to be an invisible medium that carried light waves, called the 'ether'. Experiments were designed to find it.

Consider the aether frame of reference S (inertial rest frame). [Compare this with sound waves travelling through air. The frame of reference in which the mean position of the air molecules is constant is the air rest frame.] Now the speed of light $c = 3x10^8$ m/s in the aether frame.

But observer at rest in S' , moving at a speed v relative to S, will measure the light speed to range between $c+v$ and $c-v$ depending on the direction of relative motion.

So speed of light (henceforth denoted by sol) is not an invariant quantity under Galilean transformations. So for e.m. there is only one (inertial) reference frame in which sol is *c* and we would have a way of determining a 'true' rest frame. That is, if the aether exists then it defines an absolute rest frame.

So we seem to have a 'Galilean' Principle of Relativity that applies to the Newtonian laws of mechanics but not to Maxwell's laws of electromagnetism. We are faced therefore with three possibilities:

1. Thanks to e.m., there is a preferred reference frame which we could locate experimentally.
2. A Principle of relativity exists for both Newtonian mechanics and Maxwell's e.m. and therefore Maxwell's e.m. must be wrong and needs modifying.
3. A Principle of Relativity exists for both Newtonian mechanics and Maxwell's e.m. But since the Galilean transformations only apply to Newtonian mechanics they must be wrong and a new set of transformation equations is needed that applies to both.

Einstein showed in 1905 that option 3 was the correct one.