Why is lead so kinky?

Charge Radius Isotope Shift Across the N=126 Shell Gap

1. Nuclear sizes and shapes
2. Nuclear radii: experiments
3. Nuclear radii: theory
4. Kink in lead isotopes
Shapes and sizes

- A fundamental human impulse is to find shapes
Sizes in microscopic matter

• What are the shapes or sizes of microscopic objects?
• Becomes a bit harder to find out!

Geiger, Mardsen & Rutherford experiment
Nuclear shapes

**Oblate**

$\beta < 0$

**Spherical**

$\beta = 0$

**Prolate**

$\beta > 0$

**Shape affects:**

- Single-particle spectra (Nilsson)
- Collective spectra (rotational and vibrational bands)
- Radii

**Deformation effect on charge radii**

$$\langle r^2 \rangle = \langle r^2 \rangle_{\text{sph}} \left(1 + \frac{5}{4\pi} \langle \beta_2^2 \rangle + \langle \beta_3^2 \rangle + \cdots \right) + 3\sigma^2$$
Shapes can coexist!

- Excited nuclear states can have different shapes
- Rotational bands carry imprints of such shapes

Potential energy surface of $^{186}$Pb

Nuclear radii
Droplet model and beyond

Nuclear density systematics

Charge radius

\[
\langle r_{ch}^2 \rangle = \frac{\int dr \, r^4 \rho_{ch}(r)}{\int dr \, r^2 \rho_{ch}(r)} = \frac{\int dr \, r^4}{\int dr \, r^2} = \frac{3}{5} R_{ch}^2
\]

Charge radius isotope shift

\[
\delta \langle r_{ch}^2 \rangle = \langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_{A'} \sim 0.575 \frac{\delta A}{A^{1/3}}
\]
Far from reality...

Medium mass nuclei systematics

- Kinks are ubiquitous
- Shell effects influence radii
- $1/3$ power valid in specific cases

\[ \langle r_{\text{ch}} \rangle_{\text{emp}} \sim A^{0.003Z} \]
Nuclear experiments in the 2010s

Segré Chart

- $\tau_{1/2} < 0.1$ s
- $0.1$ s $< \tau_{1/2} < 3$ s
- $3$ s $< \tau_{1/2} < 2$ mins
- $2$ mins $< \tau_{1/2} < 1$ hour
- $1$ hour $< \tau_{1/2} < 1$ day
- $1$ day $< \tau_{1/2} < 1$ year
- $1$ year $< \tau_{1/2} < 1$ Gy
- $\tau_{1/2} > 1$ Gy

~3200 isotopes

Nuclear Chart in 1966

Less than 1000
Nuclear experiments in the 2010s

Segré Chart

~3200 isotopes

Thoennessen & Sherrill, Nature (Comment) 473, 25 (2011)
Nuclear radii: experiments

4 methods to extract radii from experiments:

1. Transition energies in muonic atoms
   
   \[ a_\mu = \frac{\hbar}{m_\mu c_\alpha} = \frac{m_e}{m_\mu} a_0 \sim 200 \text{ fm} \]
   
   \[ E \sim B_{k,\alpha} = \int dr \, r^k \rho(r) e^{-\alpha r} \]

2. Elastic electron scattering

\[ \langle r_{\text{ch}}^2 \rangle = \frac{\int dr \, r^4 \rho_{\text{ch}}(r)}{\int dr \, r^2 \rho_{\text{ch}}(r)} \]

3. X-ray isotope shifts

4. Optical isotope shifts

Isotope shifts
Atoms meet nuclei

Isotope A = Z + N  Isotope A' = Z + N'

Atomic spectra

Isotope shifts

\[ \delta \nu_{i}^{A,A'} = M_{i} \frac{A' - A}{AA'} + F_{i} \delta \langle r^2 \rangle^{A,A'} \]

- Mass, \( M_{i} \), and field, \( F_{i} \), shifts obtained theoretically or empirically
- Isotope shift separation is possible \( \Rightarrow \) proliferation issues

Laser spectroscopy in unstable beams

Polonium isotopic shifts

\[ T_{1/2} = 3 \text{ mins} \]

\[ T_{1/2} = 3 \text{ yrs} \]

\[ 6p^37s^5S_2 \rightarrow 6p^37p^5P_2 \]

\[ T_{1/2} = 33 \text{ ms} \]

**EDF framework**

- Establish **energy density functional** (Skyrme EDF)

\[
\mathcal{E}_T = C_T^\rho \rho_T^2 + C_T^{\Delta \rho} \Delta \rho_T^2 + C_T^\tau \rho_T \tau_T + C_T^J \mathcal{J}_T^2 + C_T^{\nabla J} \rho_T \nabla \cdot \mathbf{J}
\]

\[
\rho(r, r') = \sum_\alpha \phi_\alpha(r) \phi_\alpha^*(r') \quad \rho_T(r) = \sum_{\sigma \tau} \rho(r \sigma \tau, r \sigma \tau) \tau^T \quad \tau_T(r) = \nabla \cdot \nabla' \rho_T(r, r')|_{r=r'}
\]

\[
\mathcal{J}_T(r) = \frac{i}{2} (\nabla' - \nabla) \rho_T(r, r')|_{r=r'} \quad \mathbf{J} = \sum_{i j k} \epsilon_{i j k} \mathcal{J}_{j k} e_i
\]

- **Solve Kohn-Sham equations** (+ BCS)

\[
h_{i j} = \frac{\delta \mathcal{E}}{\delta \rho_{i j}} \Rightarrow h_{i j} \phi_\alpha = \varepsilon_\alpha \phi_\alpha
\]

- **Use density to compute energy**

\[
\rho(r, r') = \sum_\alpha \phi_\alpha(r) \phi_\alpha^*(r') \Rightarrow \mathcal{E}(\rho)
\]

**Observables:** densities, energies, deformations
Nuclear structure with EDF
Massive parallel calculations

― Nuclear dripline statistics ― from EDF calculations

6900 ± 500 predicted isotopes

Nuclear radii
Experiments

Light nuclei systematics

Heavy nuclei systematics

\[ \langle r_{ch} \rangle_{\text{RMS}} \sim A^{1/3} \]

Isotope shift in droplet model

\[ \delta \langle r_{ch}^2 \rangle = \langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_{A'} \sim 0.575 \frac{\delta A}{A^{1/3}} \]

The lead isotopic chain

Z=82

N=126
Nuclear radii from EDF

“Isotope shifts” in semi-magic nuclei from EDF calculations

Bender, Heenen, Reinhard, Rev. Mod. Phys. 75 121 (2003)

Observation: most Skyrme forces do not reproduce kink!
Previous proposal

- **Skyrme force yields spin-orbit term:**

  \[ W_{\text{SHF}} = b_4 (\nabla \rho + \nabla \rho_n) \]

- **Relativistic EDF yields:**

  \[ W_{\text{RMF}} = \frac{\hbar^2}{(2m - C\rho)^2} C \nabla \rho \]

- **Different isospin dependence? Try richer alternative:**

  \[ W_{\text{SHF}} = b_4 \nabla \rho + b'_4 \nabla \rho_n \]
• Position of $2g_{9/2}$ relevant
• This state is affected by SO
• When less bound, sp radius is larger
• Pull on protons (via symmetry energy) should be larger
• Charge radius larger when $2g_{9/2}$ less bound
Quadrupole correlations?

Beyond mean-field calculations of isotope shifts

Observation: correlations do not affect kink mass region

Skyrme forces analysis

Symmetry energy for 16 “good” forces

- Systematic analysis ⇒ 240 Skyrme forces
- 11 macroscopic NM constraints ⇒ only 16 forces pass test
- Microscopic NM constraints ⇒ only 5 forces pass test
- How do they perform in nuclei?
Isotope shifts in lead isotopes: theory vs experiment

Isotope shifts

\[ \delta \langle r_{\text{ch}}^2 \rangle = \langle r_{\text{ch}}^2 \rangle_A - \langle r_{\text{ch}}^2 \rangle_{208} = m(A - 208) \]

\[ m_{\text{LDR}} = 0.0972 \, \text{fm}^2 \]
Single-particle spectrum of $^{210}\text{Pb}$ around Fermi surface

NRAPRii has $b_4 = b_4'$
SLy4mod has $b_4 \neq b_4'$

$1i_{11/2}$ plays an important role!
Single-particle isotope shifts

\[
\langle r_{\text{ch}}^2 \rangle_{nlj}^A = \frac{\int dr \, r^4 \left| \phi_{nlj}(r) \right|^2}{\int dr \, r^2 \left| \phi_{nlj}(r) \right|^2}
\]

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Kink in deeply bound states \( \Leftrightarrow l_{i11/2} \) is occupied
Further proof

- Neutron density changes mostly at surface
- Proton density change also has interior component
- But $1i_{11/2}$ is $\sim 1$ fm more bound than $2i_{9/2}$
Definite proof

Radial overlaps

$$\langle \pi, nl_j | \nu, 1_{11/2} \rangle = \int dr \ r^2 \phi^*_{nl_j}(r) \phi_{1_{11/2}}(r)$$

Proton-neutron overlaps in $^{208}$Pb

[Graph showing absolute radial overlaps for various proton-neutron pairs, including $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$.]
**Same thing in Polonium!**

**Isotope shifts in polonium isotopes**

Isotope shifts

\[ \delta \langle r_{\text{ch}}^2 \rangle = \langle r_{\text{ch}}^2 \rangle_A - \langle r_{\text{ch}}^2 \rangle_{210} \]
Conclusions

- Reproduction of isotope shift by and large determined by occupation of $\textit{li}_{11/2}$ neutron orbital
- This $n=1$ orbital has larger overlap with deeply bound proton orbitals
- Provides larger pull to protons via symmetry energy
- Mechanism general around $N=126$
Future work

• Explore **validity of mechanism** in other N=126

• Explore other **mass regions and kinks:**
  • Tensor in Ca isotopes?
  • Deformation in Hg?
  • Isotone shifts?

• Phil’s thesis: **dipole response** with TDHF
Thank you!

**Neutron Stars**
Nuclear Physics, Gravitational Waves & Astronomy

29-30 July 2013

Institute of Advanced Studies, University of Surrey
http://www.ias.surrey.ac.uk/workshops/neutstar/

a.rios@surrey.ac.uk