STUDY OF THE SINGLE-PARTICLE PROPERTIES OF UNSTABLE NUCLEI WITHIN THE DISPERSIVE OPTICAL MODEL

Klimochkina A.A., klimann16@gmail.com
Bespalova O.V., besp@sinp.msu.ru
Some properties of neutron-rich Ca, Zr nuclei

- New data on the properties of neutron-rich Ca nuclei

\( ^{60}\text{Ca} \)  

RIKEN, Japanese

- New magic number \( N = 32, 34 \)

\( ^{34}\text{Ca} \)  

- Deformation of Zr isotopes with \( 98<A<116 \)

Neutron thickness skin: $\Delta R_{np} = R_n - R_p$

Properties of halo nuclei
- Small (less than 2 MeV) nucleon separation energy $S_n$
- Valence nucleon is in a state with a small $l$
- Other
Application of the dispersive approach

- Mean-field theories (HFB with Skyrme, Gorny interaction)
- Shell models
- “Ab initio” (for light nuclei)

✅ Tensor force
✅ Deformation
✅ Spin-orbit interaction depends on density.

Dispersive optical model


- Mean field of dispersive optical model (DOM) is complex:
  \[ U(r, E) = V(r, E) + iW(r, E) \]

- \( V(r, E_{nlj}) \) is the real part of the dispersive optical potential (DOP)
  \[ V(r, E) = V_{HF}(r, E) + \Delta V(r, E) \]

- The dispersion relation
  \[ \Delta V_{s(d)}(r, E) = (E_F - E) \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{W_{s(d)}(r, E')}{(E' - E_F) \cdot (E - E')} dE' \]
Dispersive optical models

2) E. Sh. Soukhovitski˜ı, R. Capote,2,* J. M. Quesada et al, PHYS. REV. C 94, 064605 (2016)

Scientific reserve

- **Density distributions of** $^{90}$Zr, $^{208}$Pb, $^{116-124}$Sn nuclei Беспалова О.В., Бобошин И.Н., Варламов В.В., Ишханов Б.С., Романовский Е.А., Спасская Т.И. в журнале Известия Российской академии наук. Серия физическая, том 65, № 11, с. 1553-1557

- **Calculation of charge(proton)and matter(neutron) density distributions of Ca nuclei including those located far from the beta-stability valley.** Russian Foundation for Basic Research (project no. 16-32-00388 mol_a)

Dispersive optical potential

- \( V (r, E_{nlj} ) \) is the real part of the dispersive optical potential (DOP)

\[
-V(r, E_{nlj} ) = V_R(E_{nlj}) f(r, r_v, a_v) + 2V_{so}(E_{nlj}) \frac{1}{r} \frac{df(r, r_{so}, a_{so})}{dr} - V_C(r)
\]

- The central part of the DOP

\[
U_p(r, E) = V_{HF}(r, E) + \Delta V(r, E) + iW_I(r, E) = V_{HF}(E) f(r, r_{HF}, a_{HF}) + \Delta V_s(E) f(r, r_s, a_s) - 4a_d \Delta V_d(E) \frac{d}{dr} f(r, r_d, a_d) +
\[+iW_s f(r, r_s, a_s) - i4a_d W_d \frac{d}{dr} f(r, r_d, a_d).
\]

- The spin-orbit potential:

\[
U_{so}(r, E) = 2V_{so}(E) \frac{1}{r} \frac{d}{dr} f(r, r_{so}, a_{so})|I_s
\]

- \( V_C(r) \) is the Coulomb potential (for protons), which is usually taken to be that of a uniformly charged sphere of radius \( R_C = r_C A^{1/3} \)
Imaginary part of DOP

- Parametrization of $W(r, E)$:

  \[ J_{i,s}(E) = \alpha_i \frac{(E - E_0)^4}{(E - E_0)^4 + \beta_{i,s}^4} \]

  \[ J_d(E) = J_I(E) - J_s(E) \]

- Mass models: AME16, HFB24, KTUY05.

- Fermi energy:

  \[ E_F = -1/2(S_i(A) + S_i(A+1)) \]

Imaginary part of DOP

- Accounting for non-locality of the imaginary part

\[
E < E_F - E_a \\
W_v^{\text{neloc}}(E) = W_v(E) \left[ 1 - \frac{(E_F - E - E_a)^2}{(E_F - E - E_a)^2 + E_a^2} \right]
\]

\[
E > E_F + E_a \\
W_v^{\text{neloc}}(E) = W_v(E) + \alpha_v \left[ \sqrt{E} + \frac{(E_F + E_a)^{3/2}}{2E} - \frac{3}{2} \sqrt{E_F + E_a} \right]
\]
Real part of DOP

- Parameterization real part of DOP

\[ V_{HF}(E) = V_{HF}(E_F) \exp \left[ -\gamma(E - E_{F}) \right] \]

- Condition of agreement between summed number of nucleons in bound states and \( N(Z) \) number of the isotope.

\[ N_{n(p)} = \sum (2j + 1) N_{nlj}^{n(p)} \]

- Schrodinger equation for bound states

\[ \left[ -\nabla^2 + V(r, E_{nlj}) \right] \Phi_{nlj}(r) = E_{nlj} \Phi_{nlj}(r) \]

The occupation probabilities

\[ N_{nlj}^{n(p)}(E_{nlj}^{DOP}) = \frac{1}{2} \cdot \frac{1 - \left( \frac{E_{nlj}^{DOP} - E_F}{\sqrt{(E_{nlj}^{DOP} - E_F)^2 + (\Delta)^2}} \right)} \]

- Pairing gap parameter

\[ \Delta = -\frac{1}{4} \left\{ S_i(A+1,Z) - \frac{1}{2} S_i(A,Z) + S_i(A-1,Z) \right\} \]

For \( E_{nlj} < E_F \)

\[ N_{nlj} = 1 - \int_0^\infty \tilde{u}_{nlj}^2(r) \left\{ m_{HF}^* / m(r, E_{nlj}) \right\}^{1/2} \pi^{-1} \int_{E_{nlj}}^\infty \frac{W(r, E')}{(E' - E_{nlj})^2} dE' \] \[ dr \]

For \( E_{nlj} > E_F \)

\[ N_{nlj} = \int_0^\infty \tilde{u}_{nlj}^2(r) \left\{ m_{HF}^* / m(r, E_{nlj}) \right\}^{1/2} \pi^{-1} \int_{E_{nlj}}^{E_F} \frac{W(r, E')}{(E' - E_{nlj})^2} dE' \] \[ dr \]
$^{40}\text{Ca}$, scattering data
$^{208}$Pb, scattering data

Differential elastic scattering cross sections calculated with DOP for the system a) $n^{+208}$Pb
b) $p^{+208}$Pb

Neutron total interaction (a) and proton total reaction (b) cross-sections for $^{208}$Pb
Density distribution of stable nuclei

\[ \rho_p(n)(r) = \frac{1}{4\pi} \sum_{n,l,j} (2j + 1) N_{nlj} U_{nlj}^2(r) \]

\[ \rho_{ch}(r) = (\pi a^2)^{-3/2} \int \rho_p(r') \exp\left[-(r - r')^2/a^2\right] dr' \]

\[ <r^2>^{1/2} = \left[ \frac{\int_0^\infty r^4 \rho(r) dr}{\int_0^\infty r^2 \rho(r) dr} \right]^{1/2} \]

<table>
<thead>
<tr>
<th></th>
<th>( r_{ch}^{эксп} )</th>
<th>( r_{ch}^{ДОМ} )</th>
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<tbody>
<tr>
<td>( ^{40}\text{Ca} )</td>
<td>3.4770(0.0012)</td>
<td>3.42</td>
</tr>
<tr>
<td>( ^{42}\text{Ca} )</td>
<td>3.5086(0.0013)</td>
<td>3.48</td>
</tr>
<tr>
<td>( ^{44}\text{Ca} )</td>
<td>3.5182(0.0013)</td>
<td>3.51</td>
</tr>
<tr>
<td>( ^{46}\text{Ca} )</td>
<td>3.4956(0.0013)</td>
<td>3.49</td>
</tr>
<tr>
<td>( ^{48}\text{Ca} )</td>
<td>3.4772(0.0013)</td>
<td>3.44</td>
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</tbody>
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Fragmentation widths for $^{208}\text{Pb}$

Spectroscopic factors of neutron states for Ca isotopes

$$\Gamma_{nlj} = \frac{2 \langle W_{nlj}(E_{nlj}) \rangle}{\langle m^*_{nlj} / m \rangle}$$

$$m^*(r, E) = m \left(1 - \frac{d}{dE} V(r, E)\right)$$

$$S_{nlj} = \int_0^\infty \left[ \bar{u}_{nlj}^2(r) \left( m / m(r, E_{nlj}) \right) \right] dr$$
Neutron single-particle energies, Ca isotopes

EXP. DATA: I. N. Boboshin, Magic numbers and evolution of the shell structure of atomic nuclei, Doctor thesis (Lomonosov Moscow State University, Scobeltsyn Institute of Nuclear Physics, 2010)
Neutron single-particle energies, Zr isotopes
Root mean square radii and neutron skin

\[ \langle r_n^2 \rangle = \frac{\int_0^\infty r^4 \rho(r) \, dr}{\int_0^\infty r^2 \rho(r) \, dr} \]^{1/2} \]

Neutron skin thickness:

\[ \Delta r_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p} \]

Neutron density Ca and Zr isotopes
Neutron-rich Ca isotopes

\[ N_{h}(^{68}\text{Ca}) = 1.7 \]
\[ N_{h}(^{70}\text{Ca}) = 1.9 \]

Root-mean-square radius for nli-state

\[ R_{nlj}^{\text{rms}} = \left[ \int_{0}^{\infty} \bar{u}_{nlj}^2(r) r^4 dr \right]^{1/2} \]

<table>
<thead>
<tr>
<th>nlj</th>
<th>R_{nlj} (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2d_{5/2}</td>
<td>9.0</td>
</tr>
<tr>
<td>2d_{3/2}</td>
<td>9.1</td>
</tr>
<tr>
<td>3s_{1/2}</td>
<td>11.9</td>
</tr>
<tr>
<td>1g_{9/2}</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Density distribution for $^{70}\text{Ca}$
Neutron-rich Zr isotopes
Neutron density of $^{128}$Zr nucleus

$$\sigma_{nlj} = \frac{4\pi r^2}{(2j + 1) \rho_{nlj}}$$

\[
\begin{align*}
\left\{ \begin{array}{l}
 r_0 > r_{\text{max}}, \\
 \frac{\partial^2 \log_{10} \rho(r)}{\partial r^2} \bigg|_{r=r_0} = \frac{2}{5} \frac{\partial^2 \log_{10} \rho(r)}{\partial r^2} \bigg|_{r=r_{\text{max}}},
\end{array} \right.
\end{align*}
\]

\[
N_{\text{halo}} = 4\pi \int_{r_0}^{+\infty} \rho^n(r) r^2 dr.
\]
Summary

1. Good agreement with the available experimental data for the stable nuclei was achieved by unified manner.

2. DOM calculation predicts a sharp increase of the neutron radii of Ca and Zr isotopes far away from the $\beta$-stability valley.

3. The obtained results suggest that the neutron halo structure forms in neutron rich Ca and Zr nuclei in the vicinity of the neutron drip line.
   ✓ Single-particle states contributing to the halo are determined.
   ✓ The number of nucleons in halo states is estimated.
   ✓ The obtained results are in qualitative agreement with the calculations by the Hartree-Fock-Bogolyubov relativistic model with the continuum.
   ✓ Halo states of Ca and Zr isotopes has low orbital moment $l$. The state located below is characterized by high orbital moment $l$.

4. The present study that demonstrates DOM is a powerful tool to predict single-particle characteristics of nuclei far away from the $\beta$-stability valley.
Thank you for attention!