In this course, I will outline some of the methods that are now being used to build bridges between the knowledge of nuclear interaction that has been gathered over the last 80 years and quantum chromodynamics, the fundamental theory of the strong interaction.

There are two ingredients in the modern approach to nuclear forces. The first is the chiral symmetry of QCD, which reflects the fact that it is a theory with almost massless up and down quarks. As a result pions are much lighter than all other hadrons and play a special role in nuclear physics.

A further consequence of chiral symmetry is that the interactions between pions and nucleons are weak at low energies. As a result we can make a perturbative expansion not in terms of some small coupling but in powers of momenta and pion masses. This expansion in low-energy scales is the starting point for chiral effective field theories of nuclear forces.

More extensive lecture notes on some of these topics can be found in:

- TALENT/INT Course on nuclear forces (2013), by R. Furnstahl (Ohio State) and A. Schwenk (Darmstadt)
- Quarks, nuclei and the cosmos, by X. Ji (Maryland)
  http://physics.umd.edu/courses/Phys741/xji/index.htm

My recommendation for a book on nuclear physics at this level is:


The newer topics of effective field theory are covered in the (much more theory-oriented) book:


The exercises below are intended to get you thinking about the ideas involved, without getting into too much mathematical detail.

**Lecture 1**

1. List all the allowed quantum numbers $J^{PC}$ for quark-antiquark states with total angular momentum $J = 0, 1$ and $2$. Which quantum numbers are not possible for $q\bar{q}$ states and so must be exotic?
2. The $$\pi_1(1400)$$ meson has been observed to decay into a $$\pi^0$$ and an $$\eta^0$$ with relative angular momentum $$L = 1$$. The $$\pi^0$$ and $$\eta^0$$ mesons both have $$J^{PC} = 0^{-+}$$. Deduce the quantum numbers $$J^{PC}$$ of the $$\pi_1(1400)$$ and comment on your result.

Lecture 2

1. What are the cyclotron frequency $$\omega_c$$ and the spin-precession (Larmor) frequency $$\omega_s$$ for a spin-$\frac{1}{2}$ particle of mass $$m$$, charge $$e$$ and gyromagnetic ratio $$g$$ moving in a magnetic field $$B$$? Use these to explain why the helicity of a particle with $$g = 2$$ is preserved in a magnetic field. Is helicity still preserved when a massive particle moves in an electric field? What happens if the particle is massless?

2. The $$q\bar{q}$$ pairs that condense in the QCD vacuum have $$J = 0$$ and consist of either a right-handed quark and a right-handed antiquark, or a pair where both are left-handed. A right-handed antiquark is the antiparticle of a left-handed particle. (Think about the neutrinos and antineutrinos emitted in $$\beta$$ decays.) What can happen to a left-handed quark travelling in this vacuum? What does this imply about the quark?

Lecture 3

1. What are the allowed values of the orbital angular momentum $$L$$ for two neutrons with total spin $$S = 0$$?

2. A proton and a neutron are in a $$^3G_3$$ state. Are there other states with same total $$J$$ and parity that could mix with this one? Are there $$pp$$ and $$nn$$ states with the same quantum numbers?

3. For a system of two nucleons, find the eigenvalues of $$\tau^{(1)} \cdot \tau^{(2)}$$.

[Hint: use $$\hat{\tau}^{(i)} = \frac{1}{2} \tau^{(i)}$$ and think about the square of the total isospin $$\hat{T} = \hat{\tau}^{(1)} + \hat{\tau}^{(2)}$$.

4. The wave function of an $$S$$-wave bound state in the square-well potential of depth $$V_0$$ and radius $$R$$ has the form

$$u(r) = \begin{cases} A \sin(Kr) & \text{for } r < R \\ Be^{-\gamma r} & \text{for } r > R \end{cases},$$

where the energy of the bound state is $$E = -\gamma^2/M$$ and $$K^2 = K_0^2 - \gamma^2$$.

By matching $$u$$ and $$u'$$ at $$r = R$$, find an equation for $$\gamma$$. Show that for a very shallow bound state, i.e. with $$|E| \ll V_0$$, $$\gamma$$ can be related to the scattering length $$a$$ for this potential. How does this relationship depend on the radius $$R$$ in the case where $$a \gg R$$?
Lecture 4

1. Use the result of the previous question to show that the energy of a shallow bound state is related to the scattering length by \( E = -1/2Ma^2 \).

For the \(^3S_1\) np wave, the scattering length is \( a = 5.4 \text{ fm} \). Use this to estimate the binding energy of the deuteron and compare your answer with the observed binding energy (2.224 MeV).

2. Evaluate the expectation value of the tensor operator
\[
\hat{S}_{12} = \frac{3}{r^2} (\sigma^{(1)} \cdot r)(\sigma^{(2)} \cdot r) - \sigma^{(1)} \cdot \sigma^{(2)}.
\]
in the state \( \psi_{11} \) where the two nucleons have total spin \( S = 1 \) and \( M_S = +1 \). Show that your answer is proportional to \( Y_{20}(\theta, \phi) \). What does this imply for the orbital matrix elements of this operator?

3. Show that the leading four-nucleon force appears at order \( Q^4 \) in the framework of Weinberg’s power counting.

[Hint: Consider four pions interacting at a contact interaction. Include factors for the phase space of two extra nucleons compared to the NN force.]