Fundamentals of Gamma-Ray Spectroscopy

(a) Introduction
(b) Gamma-ray interactions in matter
(c) Gamma-ray detection
(d) Large arrays and their properties
(e) High-fold coincidence analysis
(f) Spectroscopic techniques for probing nuclear structure

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What is Gamma-ray Spectroscopy

Definition from Wikipedia

Most radioactive sources produce gamma rays, which are of various energies and intensities. When these emissions are detected and analyzed with a spectroscopy system, a gamma-ray energy spectrum can be produced. A detailed analysis of this spectrum is typically used to determine the identity and quantity of gamma emitters present in a gamma source, and is a vital tool in radiometric assay. The gamma spectrum is characteristic of the gamma-emitting nuclides contained in the source, just as in optical spectroscopy, the optical spectrum is characteristic of the material contained in a sample.

The equipment used in gamma spectroscopy includes an energy-sensitive radiation detector, electronics to process detector signals produced by the detector, such as a pulse sorter (i.e., multichannel analyzer), and associated amplifiers and data readout devices to generate, display, and store the spectrum. Other components, such as rate meters and peak position stabilizers, may also be included. The most common detectors include sodium iodide (NaI) scintillation counters and high-purity germanium detectors.
What is Gamma-ray Spectroscopy

An analytical method for the measurement of the intensities and energies of gamma radiation. Scintillation or semiconductor radiation detectors coupled to various types of electronic circuitry, enable a spectrum to be accumulated. This may be used to identify the gamma-emitting radioisotopes, and their energy and intensities and be used to determine the corresponding element concentrations.


What can we measure?

\[ I_i = E_i \]
\[ I_f = E_f \]
\[ E_r = E_i - E_f \]
\[ |I_i - I_f| \leq L \leq I_i + I_f \]

\[ \Delta \pi(EL) = (-1)^{+} \]
\[ \Delta \pi(ML) = (-1)^{L+1} \]

\[ T_{f \to i}(\sigma \lambda) = \frac{2(L+1)}{\varepsilon_0 L [(2L+1)!]^2} \left( \frac{E_y}{C} \right)^{2L+1} \left| \langle \psi_f | M(\sigma \lambda) | \psi_i \rangle \right|^2 \]

Reduced transition probability \( B(\sigma \lambda) \)
What can we learn from these measurements?

Transition energies and coincidence relationships
  Level structure

Angular correlations and linear polarisations
  Spin, Parity (also magnetic moments).

Doppler shift / lineshapes
  Lifetimes, quadrupole moment

Branching ratios, mixing ratios
  Wavefunctions, transition matrix elements, etc

Why study gamma-ray emissions?

Gamma rays provide a superb probe for nuclear structure!

Relatively easy to detect with good efficiency and resolution.

Emitted by almost all low-lying states.

Penetrating enough to escape from target chambers to reach detectors.

Gamma rays arise from EM interactions and allow a probe of structure without large perturbations of the nucleus.

No model dependence in the interaction (EM is well understood).
Why study gamma-ray emissions? (Physics)

Shell Structure
Stability of shell closures; New shells; Levels near Fermi surface; Isospin dependence; Deformation.

Collective Modes
Rotation; Vibration; Magnetic rotation; Wobbling, Giant resonances.

Nuclear Shapes
Octupole shapes; Shape coexistence; Triaxiality; Superdeformation.

High-Spin States
Band termination; Coriolis forces; Fission stability;

... and many many more!

Out-of-beam Gamma-ray Spectroscopy

Out-of-beam spectroscopy concerns delayed emission of gamma-ray from nuclear decays. Gamma-ray multiplicity is usually low.
Gamma rays emitted from stopped nucleus – excellent energy resolution.

Radioactive sources:
Fission fragments e.g. Cf sources

Stopped radioactive ion beams:
ISOL
Fragmentation or spallation products

De-excitation of isomeric states
Case Study: $\gamma$-ray emissions from stopped fragments

- Removal of nucleons from heavy projectile in peripheral collisions
- Hot participation zone
- Projectile
- Target
- Projectile
- Fragment
- Cooling by evaporation
- Projectile
- Fragment

Case Study: The RISING stopped beam campaign

- FRS provides secondary radioactive ion beams:
  - fragmentation and fission of primary beams
  - high secondary beam energies: 100 – 500 MeV/u
  - fully stripped ions

Fragment Recoil Separator (FRS):

- Use FRS to ID individual nuclei
- Transport some isomeric states (TOF ~200ns)
- Stop and correlate decays with nuclide identification.
Identification of exotic nuclei following fragmentation reaction.


The RISING spectrometer array.

- 5 at 51°, 5 at 90° and 5 at 129° all at 209.8 mm
- Photopoint efficiency 17.2% at 1.3 MeV
- 8 BaF$_2$ detectors (185-220 mm)
OLD: 60 MeV/A $^{92}$Mo beam

NEW: 600 MeV/u $^{107}$Ag beam

Case Study: γ-ray emissions beyond the proton drip line
The GREAT Spectrometer

- 2 x 60mm x 40mm DSSD (4800 pixels)
- 28 x 28mm x 28mm PIN Diodes
- 12cm x 6cm Segmented Planar Ge
- Compton-Suppressed Segmented Ge Clover
- Position-Sensitive MWPC
The RITU gas-filled separator

Isomer de-excitations beyond the proton drip line

In-beam γ-ray spectroscopy

In-beam spectroscopy concerns the measurement of gamma-ray emissions that occur promptly at the target position. Gamma-ray multiplicity can vary from 1 to > 30.

Gamma rays emitted by moving reaction product – velocity and spread (both speed and angle) strongly affects energy resolution.

Low-energy in-beam spectroscopy (up to 10 MeV per nucleon) can be probed by various reaction mechanisms:
- Fusion evaporation
- Multi-nucleon transfer reactions
- Coulomb excitation
- Single nucleon transfer reactions

Intermediate-energy in-beam spectroscopy (50-100 MeV per nucleon) can be probed by various reaction mechanisms:
- Coulomb excitation
- Spallation
- Knockout

Non-Collective Level Scheme: $^{148}$Gd

$^{148}$Gd is an example of a nucleus showing single-particle behaviour

Complicated set of energy levels

No regular features e.g. band structures

Some states are isomeric
Collective Level Scheme: $^{174}$Hf

This nucleus has 347 known levels and 516 gamma rays!

Case study: Heavy transitional nuclei

The nucleus is one of nature's most diverse and interesting quantum many-body systems. It brings together many types of behaviour, almost all of which are found individually in other systems but which, in nuclei, interact with one another.

The major elementary excitations in nuclei can be associated with either single-particle or collective modes.

**Single particle:** nucleons are assumed to move independently in an average potential. Excited states are built on coupling small number of individual nucleons.

**Collective:** Close analogies to molecular and solid-state physics. Large number of nucleons behaving in a correlated manner.

While these modes can exist in isolation, it is the interaction between them that gives nuclear spectroscopy its rich diversity!
\[ \beta_2 = 0.161 \]
\[ \beta_2 = 0.267 \]
\[ \beta_2 = 0.018 \]

In-beam gamma-ray spectroscopy
The JUROGAM spectrometer
43 EUROGAM Phase 1 Ge detectors + BGO shields
Efficiency at 1.3 MeV ~4%
Alignments

$I_x = \sqrt{I(I+1) - K^2}$

Orbitals with different angular momenta yield different alignments!
Gamma-Ray Interactions in Matter and Detectors
**Reaction mechanisms for synthesising exotic nuclei**

(Stable and radioactive beams)

- Fusion Evaporation Reactions
- Nuclear Fragmentation
- Deep Inelastic Reactions
- Fission
- Transfer Reactions

**Fusion-evaporation reactions**

- Large cross section: ~1 barn
- Ideal for populating states with very high angular momentum (as high as ~70 \( \hbar \))
- Large gamma-ray multiplicity
  - Need high-granularity high-efficiency high-resolution gamma detector array
- No Coulomb barrier for neutrons
  - Tends towards proton-rich nuclei, hard to make neutron-rich

In-Beam Gamma Spectroscopy:
**Fusion-Evaporation Reactions**

- Preferentially populates states along the yrast line.
Gamma-ray interactions in matter

Gamma (γ) rays interact with matter via three main reaction mechanisms.

- Photoelectric absorption
- Compton scattering
- Pair Production

Photoelectric absorption

Einstein won the Nobel Prize for Physics for the discovery of the photoelectric effect.

In this mechanism a γ-ray interacts with a bound atomic electron.

The photon completely disappears and is replaced by an energetic photoelectron.

The energy of the photoelectron can be written

\[ E_e = h\nu - E_B. \]

The incident gamma-ray photon minus that of the binding energy of the electron (\( E_B = 12\text{eV} \) in germanium).
**Compton scattering**

Compton won the Nobel Prize for Physics for the discovery of the Compton effect.

The γ-ray interacts with a loosely bound atomic electron.

The incoming γ-ray is scattered through an angle θ with respect to its original direction.

The photon transfers a proportion of its energy to a recoil electron. The expression that relates the energy of the scattered photon to the energy of the incident photon is

\[ E_s = \frac{E_i}{1 + \frac{E_i}{m_e c^2} (1 - \cos \theta)} \]

**Pair production**

If the energy of a gamma-ray exceeds twice the rest mass energy of an electron (1.02 MeV) the process of pair production is possible.

A gamma-ray disappears in the Coulomb field of the nucleus and is replaced by an electron-positron pair.

The excess energy above 1.02 MeV goes to the kinetic energy of the electron and the positron.

The positron will subsequently annihilate after slowing down in the absorbing medium, producing two annihilation photons (511 keV), which may be subsequently detected.
Interactions in a small detector

A small detector is one so small that only one interaction can take place within it. Only the photoelectric effect will produce full energy absorption. Compton scattering events will produce the Compton continuum. Pair production will give rise to the double escape peak due to both gamma-rays escaping.
Interactions in a large detector

A **large detector** is one in which we can ignore the surface of the detector. Various successive photoelectric absorption, Compton scattering and pair production interactions will occur. The result is complete absorption of the gamma-ray and a single gamma-ray peak, referred to as the full energy peak.

![Diagram of interactions in a large detector](image)

Interactions in a real detector

Within a **real detector** the interaction outcome is not as simple to predict as the small or large detector case. Compton scattering may be followed by other Compton scatterings before the gamma-ray photon escapes from the detector. Also, pair production may be followed by the loss of only one annihilation gamma-ray, resulting in a single escape peak as well as a double escape peak.

![Diagram of interactions in a real detector](image)
Energy dependence of the interactions

Gamma-ray photons can have a large range of energies.

Typical energies of interest to us range between 60 keV and 10 MeV.

Other interactions in a real detector

Thomson Scattering

Low-energy coherent scattering off free electrons. Not important in the energy range concerned with most nuclear structure studies.

Nuclear Thomson Scattering

Low energy coherent scattering off nucleus. Small effect.

Dellbrück Scattering

Scattering in the Coulomb field of the nucleus. Important at $E_\gamma > 3$ MeV.
Gamma-ray detectors

Both scintillator and semiconductor γ-ray detectors are really Detecting the recoiling electrons as they scatter off other electrons in the medium.

In scintillation detectors (e.g. NaI, BGO) the scattering of electrons excites atoms, which then de-excite by emitting visible light. The light is collected in photomultiplier tubes (PMTs) where it generates a pulse proportional to the light collected.

In semiconductor detectors (e.g. Ge, CZT) the scattering of the electrons generates electron hole pairs inside a reverse-biased diode. The freed electrons and holes drift in a large electric field thereby generating a pulse that is proportional to the energy deposited. Each e-h pair requires ~3 eV. A low-noise charge integrating preamplifier is used.
Scintillators or semiconductors?

Scintillators usually have:
- Poorer resolution (e.g. NaI ~6% at 1.3 MeV)
- Higher density
- Higher Z (and therefore better efficiency).

Ge semiconductor detectors have:
- The best resolution (~0.15 at 1.3 MeV)
- Generally poorer efficiency and peak-to-total (depends on crystal size)
- Requires cryogenic operation.

Some other semiconductors (e.g. CZT) have:
- High Z (but cannot be made into large crystals.
- Poorer resolution than Ge
- Do not require cryogenics.

Escape suppressed HP Ge detector
Compton background

The Compton background is a serious problem for high-fold coincidence data.

The peak-to-total ratio for a bare Ge detectors at 1.0 – 1.5 MeV is typically ~0.2 so

- only 20% of single γ events are full energy.
- for γ-γ coincidences, only 4% of events are full energy.
- for γ-γ-γ γ coincidences, only 0.16% of events are full energy.

Something must be done to avoid collecting lots of background due to Compton scattering – **Compton suppression**.

Compton suppression uses a scintillator material to surround the Ge to detect scattered γ rays and veto the bad events. Compton suppression raises the peak-to-total to ~ 0.6.
Escape suppression increases the peak-to-total (PT) from ~20% to ~60%.

Discussion points for tutorial

What is high-fold coincidence data?

What is coincidence fold and how does it differ from γ-ray multiplicity?

Why is scintillator material used for Compton suppression?

Why is BGO usually chosen?
Large spectrometer arrays and their properties

First-generation arrays

The Spin Spectrometer
- the first 4m array

72 close-packed NaI(Tl) detectors

Capable of measuring simultaneously the gamma-ray multiplicity and the total gamma-ray energy event-by-event.

Inner radius 178 mm
Thickness 178 mm
Energy resolution ~ 6.3% at 1332 keV
Time resolution ~ 2 ns FWHM.
The Darmstadt-Heidelberg Crystal Ball

A spherical shell of 162 NaI-detectors
25 cm inner radius
20 cm crystal thickness

Second-generation arrays

Total Escape Suppressed Spectrometer Array
The 8$\pi$ Spectrometer
Chalk River, Canada
A "second-generation" array

20 Ge detectors (25% efficiency)
with BGO Compton suppressors

72-element BGO inner ball

Third-generation arrays

JUROGAM

EUROGAM @ JYFL

43 Compton suppressed Ge detectors.
Spectrometer arrays and their sensitivity

There are many gamma-ray spectrometer arrays in use around the world.

Requirements for a good spectrometer:

(a) High photopeak detection efficiency.
(b) Excellent resolution.
(c) Good peak-to-total signal.
(d) High granularity.

Spectrometers are designed for specific reaction mechanisms / applications.

e.g. High gamma-ray multiplicities and large Doppler shifts require high granularity.
Spectrometer array efficiency

The efficiency measures the ability of an array to collect statistics.

The AFRODITE spectrometer array, iThemba Laboratory

Discussion point: Why does the efficiency curve have this shape?

Resolving power

The resolving power measures the effectiveness of an array in isolating a single γ-ray sequence in a complex spectrum.

Average separation of consecutive γ rays in a cascade.

\[ R = \left( \frac{SE_\gamma}{\Delta E_\gamma} \right) PT \]  Peak-to-Total ratio

FWHM of detector resolution for γ rays in spectrum

The resolving power can only be improved by:
- Improving the peak-to-total ratio
- Improving the energy resolution of the array.
High-fold data & multidimensional analysis

\[ {}^{32}S + {}^{112}Sn \rightarrow {}^{152}Dy + 4n \]

Probability of detecting high-fold events increases with increased photopeak detection efficiency.

The limit of observation for an array \( \alpha_0 \)

In order to detect a peak two criteria must be satisfied.

(a) The peak must stand above the background
(b) The peak must be statistically significant.

The limit of observation of an array, \( \alpha_0 \), is the minimum intensity that can be detected. This is related to the photopeak efficiency and the resolving power of the array.

The peak-to-background ratio depends on resolving power by

\[
\left( \frac{N_p}{N_B} \right)_n = \alpha_0 (0.76R)^n
\]

where \([N_p/N_B]_n\) is the final peak to background when an n-1 selection has been made on the data.
A superdeformed band in $^{149}$Gd.

The availability of high-fold coincidence data allows the possibility of setting multiple 'gates' to enhance specific features.

The higher the $\gamma$-ray fold the lower the limit of observation from resolving power alone.


Sensitivity

It is assumed that the background is completely uncorrelated.

In a real spectrum the background is made up of:

- Compton scattered gamma rays.
- Statistical gamma rays.
- Feeding gamma rays.

which have coincidence relationships to the cascades.

Additional ancillary detectors could also improve the sensitivity of the array. This introduces another factor $R_0$ such that

$$\left( \frac{N_p}{N_b} \right)_n = \alpha_0 R_0 (0.76 R)^n.$$  

More on this later!
The limit of observation for an array \( \alpha \)

To satisfy the second criterion (that the peak must be statistically significant) ....

The number of counts in the peak following a cascade of \( M_\gamma \) transitions is given by

\[
N^p_n = \frac{\alpha_0 (0.76 PT)^n N_{on}}{M_\gamma (M_\gamma - 1) \cdots (M_\gamma - n + 1)}
\]

where \( N_{on} \) is the number of \( n \)-fold coincidences obtained by unpacking the measured coincidences \( \geq n \).

\( N_{on} \) in turn depends on the photopeak efficiency of the array.
Case study: Using ancillary detectors

The nucleus of interest is often weakly populated compared to a large background of other nuclei.

Additional sensitivity can be achieved by

- Charged-particle detectors
- Neutron Detectors
- Recoil Detectors
- Tagging Techniques

Neutrons are deeply bound therefore charged particle emission is favoured despite the Coulomb barrier.

Factors affecting resolution

- St telescope
  - $\frac{dE}{dx} = \frac{mZ^2}{E}$
  - 95 Ce(Tl) Scintillators in 9 rings
  - used with Gammasphere

- Microball, Washington University St. Louis

- St telescope
  - $\frac{dE}{dx}$
  - 30 hexagons
  - 12 pentagons
  - used with GASP and Euroball

- Italian Silicon Sphere ISIS
  - Laboratori Nazionali di Legnaro
Figure 2.12: Total energy (slow) against fast component for the MICROBALL from the $^{58}\text{Ni} + ^{58}\text{Ni}$ reaction studied in this work. The separation for protons, deuterons, H-3 and alpha particles is clear.


Figure 2.13: Plot of energy against zero crossover time (ZCT). Notice the excellent $\alpha$-time resolution.
Discussion point:

Why do peaks from the 3p reaction channel appear in the 1p selected spectrum?

AJ Boston,
Factors affecting array performance

The major factors affecting the final energy resolution (FWHM) of the array at a particular energy are as follows.

\[ \Delta E_{\gamma}^{\text{final}} = \left( \Delta E_{\text{int}}^2 + \Delta \theta_D^2 + \Delta \theta_N^2 + \Delta V^2 \right)^{\frac{1}{2}} \]

- \( \Delta E_{\text{int}} \) - The intrinsic resolution of the detector system. This includes contributions from the detector itself and the electronic components used to process the signal.
- \( \Delta \theta_D \) - The Doppler broadening arising from the opening angle of the detectors.
- \( \Delta \theta_N \) - The Doppler broadening arising from the angular spread of recoils in the target.
- \( \Delta V \) - The Doppler broadening arising from the velocity (energy) variation of the recoils across the target.
Doppler broadening

Broadening of detected γ-ray energy arises from:
- Spread in recoil velocity ΔV
- Distribution in the direction of recoil Δθ_N
- Detector opening angle Δθ_D

Doppler shift

\[ E_\gamma = E_\gamma^0 \sqrt{\frac{1 - \frac{V^2}{c^2}}{1 - \frac{V}{c} \cos \theta}} \]

Minimizing Doppler broadening

There are two ways in which gamma-ray spectroscopists can mitigate the effects of Doppler broadening.

- Reduce the detector opening angle (lower Δθ_D)
  - Detector granularity

- Minimise the target thickness (lower spread in recoil velocity ΔV)

Experimentalists often choose to use two or three stacked targets rather than a single thick target e.g. 2 x 0.5 mg/cm² rather than a single 1 mg/cm².

This works for normal kinematics where the spread in velocities ΔV arises from the slowing of the recoil and not the beam.
**Target thickness**
Light beam, heavy recoil

The main contribution to poor resolution arise from **Doppler effects**. Require detector granularity!

**Doppler broadening of 547 keV line from the Coulex of $^{197}$Au**

Discussion topics:
Derive Doppler broadening formula for Ge detector.
Where is Doppler broadening most problematic?
Other considerations

Other problems associated with high multiplicity events reduce the resolving power (by reducing the PT value) and the photopeak efficiency.

- Isolated hit probability (accounts for more than one γ or neutron hitting the detector).
- Scattering of γ or neutron from surrounding material.
- Probability that detector is hit first by a neutron.
Gamma-ray tracking

Without Compton suppression shields

With BGO shielding

With highly segmented detectors

\[ E' = \frac{E_0}{1 + (E_0/mc^2)(1 - \cos \theta)} \]

Compton continuum
⇒ Large peak to total ratio

Less solid angle coverage
⇒ Big drop in efficiency

Path of γ-ray reconstructed to form full energy event
⇒ Compton continuum reduced
⇒ Excellent efficiency ~50% @1MeV
⇒ Greatly improved angular resolution (~1°) to reduce Doppler effects
Ingredients of γ-ray tracking

1. Highly segmented HPGe detectors
2. Digital electronics to record and process segment signals
3. Identified interaction \((x,y,z,E,t)\)
4. Reconstruction of tracks e.g. by evaluation of permutations of interaction points

Pulse Shape Analysis to decompose recorded waves

reconstructed γ-rays

AGATA

Ge crystals size:
- length 90 mm
- diameter 80 mm

180 hexagonal crystals 3 shapes
60 triple-clusters all equal
Inner radius (Ge) 22 cm
Amount of germanium 310 kg
Solid angle coverage 80 %
Singles rate \(~50\) kHz
6480 segments

Efficiency: 40% \((M_c=1)\) 25% \((M_c=30)\)
Peak/Total: 65% \((M_c=1)\) 50% \((M_c=30)\)
Generating spectra from high-fold coincidence data.

Raw Event 1: 100, 200.
Raw Event 2: 100, 200, 300.
Raw Event 3: 100, 200, 500, 250.

Event 1 (100, 200) (200,100)
Event 2 (100,200) (200,100) (100,300) (300,100) (200,300) (300,200)
Event 3 (100,200) (200,100) (100,500) (500,100) (100,250) (250,100) (200,500) (500,200) (200,250) (250,200) (250,500) (500,250)
Can you tell the difference between these bands?

Differences revealed by angular distributions

Superdeformed Band in $^{192}\text{Hg}$
(Cascade of E2 transitions)

M1 cascade in $^{198}\text{Pb}$
Angular distributions

The general form for the angular distribution function of radiation emitted following a heavy-ion fusion-evaporation reaction is:

\[ W(\theta) = A_0 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta) + \ldots \]

or with \( a_k = A_k / A_0 \)

\[ W(\theta) = 1 + a_2 P_2(\cos \theta) + a_4 P_4(\cos \theta) + \ldots \]

where \( P_n(\cos \theta) \) are Legendre polynomials.

Modifications to \( a_k \) are made to allow for the finite size of the detectors (geometric attenuation factor \( Q_m \)) and the incomplete alignment of the magnetic substates \( (a_{k_{\text{max}}}) \).

The measured \( a_k \) coefficients are compared to theory for different types of radiation.

Angular distributions

It is possible to calculate \( a_{k_{\text{max}}} \) values, see

E. der Mateosian & A.W. Sanger,

If the two lowest multipoles of the radiation are \( L \) and \( L' = L + 1 \),
the \( a_k \) coefficients may be written as:

\[ A_n / A_0 = a_n B_n(J_l) \left[ 1/(1+\delta^2) \right] \left[ F_n(J_l L L' J_l) + 2\delta F_n(J_l L L' L J_l) + \delta^2 F_n(J_l L L' L' J_l) \right] \]

where \( a_n \) are attenuation coefficients, \( B_n(J_l) \) are statistical tensors for complete alignment, and \( \delta \) is the multipole mixing ratio.
The mixing ratio

The mixing ratio is a ratio of reduced matrix elements and so the phase is a measurable quantity, i.e. it can be positive or negative.

\[ \delta = \frac{\langle I_f | L' | I_i \rangle}{\langle I_f | L | I_i \rangle} \]

\[ \delta_{E2/M1}' = \frac{|\langle I_f | E2 | I_i \rangle|^2}{\langle I_f | M1 | I_i \rangle} = \frac{T(E2: I \rightarrow I-1)}{T(M1: I \rightarrow I-1)} \]

Angular distributions

Typically:

Pure stretched quadrupole

\[ a_2 \sim +0.35, \ a_4 \sim -0.05 \]

Pure stretched dipole

\[ a_2 \sim -0.3, \ a_4 \sim 0 \]

Caution: unstretched pure dipoles (\( \Delta I=0, E1 \)).

Linear polarisation

Compton scattering can be used to measure the gamma ray linear polarisation – the direction of the electric vector with respect to the beam-detector plane.

The linear polarisation distinguishes between magnetic (M) and electric (E) character of radiation of the same multipolarity.

The scattering cross section is larger in the direction perpendicular to the electric field vector of the radiation.

Linear polarisation

The experimental asymmetry is defined as:

\[ A = \frac{N_{90} - N_0}{N_{90} + N_0} \]

where \( N_{90} \) and \( N_0 \) are the intensities of scattered photons perpendicular and parallel to the reaction plane.

The experimental linear polarisation is then:

\[ P = \frac{A}{Q} \]

where \( Q \) is the polarisation sensitivity of the detector (a function of gamma ray energy).

For a stretched E2: \( P > 0 \)  For a stretched M1: \( P < 0 \)
Spins and parities

Combining linear polarisation and angular correlation measurements uniquely defines the multipolarity of gamma rays.

Data from Eurogam

Lifetimes & quadrupole moments

Deformed shell gaps (new ‘magic numbers’) emerge when the ratio of the major and minor nuclear axes are equal to the ratio of small integers.

A superdeformed shape has a major to minor axis ratio of 2:1.

A hyperdeformed shape has a major to minor axis ratio of 3:1.
Lifetimes & quadrupole moments

The SD band in $^{152}$Dy is a very regular structure with equally spaced gamma-ray transitions.

The spacing is relatively small, i.e. the band has a large moment of inertia (close to the rigid body value).

Original SD γ-ray spectrum from 1986 (Daresbury)

Lifetimes & quadrupole moments

$$\frac{1}{\tau} = \frac{T(E2)}{B(E2)} = 1.23 \times 10^{13} E_\gamma^5 B(E2)$$

$$B(E2 : I \rightarrow I - 2) = \frac{5}{16\pi} \langle I \ 2 \ K \ 0 | I - 2 \ K \rangle^2 Q_t^2$$

$$\frac{1}{\tau} = 1.223 \times 10^{12} E_\gamma^5 \langle I \ 2 \ K \ 0 | I - 2 \ K \rangle^2 Q_t^2$$

Units: $T(E2)$ in s$^{-1}$, $B(E2)$ in (eb)$^2$, $E_\gamma$ in MeV.
### Timescale for Lifetime Measurement Techniques

<table>
<thead>
<tr>
<th>$\tau$ (s)</th>
<th>$\Gamma$ (eV)</th>
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<tbody>
<tr>
<td>$10^{-15}$</td>
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- **Indirect Methods**
  - (Measure $\tau$)
  - Electronic Timing
  - RDM
  - DSAM
  - GRID

- **Indirect Methods**
  - (Measure $\Gamma$)
  - Resonance Fluorescence
  - Direct Width Measurements
  - Coulex

### Doppler shift attenuation method

Measure lifetimes in the range $10^{-15} < \tau < 10^{-12}$ s.

Stopping time in metal foil is comparable to lifetimes of excited states.

$$E_{\text{shifted}} = E_{\text{true}} \left(1 + \frac{V}{c} \cos \theta\right)$$
Centroid shift method

\[ E_f = E_{\text{true}} \left(1 + \frac{\bar{v}}{c} \cos \theta_f \right) \]

\[ E_b = E_{\text{true}} \left(1 + \frac{\bar{v}}{c} \cos \theta_b \right) \]

If \( \theta = \theta_{\text{forward}} = 180 - \theta_{\text{backward}} \)

\[ E_f - E_b = 2E_{\text{true}} \frac{v}{c} \cos \theta \]

\[ F(\tau) = \frac{\bar{v}}{V_0} \text{ and } \beta = \frac{v_0}{c} \]

\[ F(\tau) = \frac{E_f - E_b}{2E_{\text{true}} \beta \cos \theta} \]

Calculate \( F(\tau) \) for different quadrupole Moments and find best fit.

- \(^{131}\text{Ce SD1} \quad Q = 7.3(3)\)
- \(^{131}\text{Ce SD2} \quad Q = 8.2(4)\)
- \(^{132}\text{Ce SD 1} \quad Q = 7.4(3)\)
- \(^{132}\text{Ce SD 2} \quad Q = 7.2(4)\)
- \(^{132}\text{Ce SD 3} \quad Q = 7.0(4)\)

Excited SD band in \(^{131}\text{Ce}\) has a larger \( Q \) due to a configuration involving a hole in neutron \( d_{5/2} \) orbital.

Specific orbitals have different core polarising properties.

RM Clark et al., PRL 76 (1996) 3510.
**Shape coexistence**

Different shapes, co-existing at low excitation energy

![Potential Energy Surface for $^{186}$Pb](image)

$^{190}$Pd

$^{186}$Pb

$^{190}$Pd

$^{186}$Pb

Potential Energy Surface for $^{186}$Pb

A. Andreyev et al., Nature 405 (2000) 410

**Tagged Recoil Distance Method**

$I_{\text{degraded}} = I e^{-d/v_{\text{r}}}$

$I_{\text{shifted}} = I (1 - e^{-d/v_{\text{r}}})$

$$\frac{I_{\text{degraded}}}{I_{\text{degraded}} + I_{\text{shifted}}} = e^{-d/v_{\text{r}}}$$

Arbitrary units

Distance [µm]

Energy [keV]
### The Kolin Plunger Device

![Diagram of the Kolin Plunger Device](image)

<table>
<thead>
<tr>
<th>(E_j) (MeV)</th>
<th>(J^\pi)</th>
<th>(r) [fm]</th>
<th>(H \cdot 22) [W.u.]</th>
<th>(Q_{\alpha \beta}) [eV]</th>
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<tbody>
<tr>
<td>604 Pb</td>
<td>2(^+)</td>
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<td>6(2)</td>
<td>1.7(2)</td>
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<td>263</td>
<td>4(^+)</td>
<td>18(4)</td>
<td>5(0) 12(0)</td>
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<tr>
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<td>415</td>
<td>8(^+)</td>
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<td>2(0) 14(0)</td>
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### Spectroscopic Properties

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<th>(J^\pi)</th>
<th>(r) [fm]</th>
<th>(H \cdot 22) [W.u.]</th>
<th>(Q_{\alpha \beta}) [eV]</th>
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<tr>
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<td>Band IV</td>
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</table>

![Chart showing spectroscopic properties](image)

**1\(^{86}\)Pb**